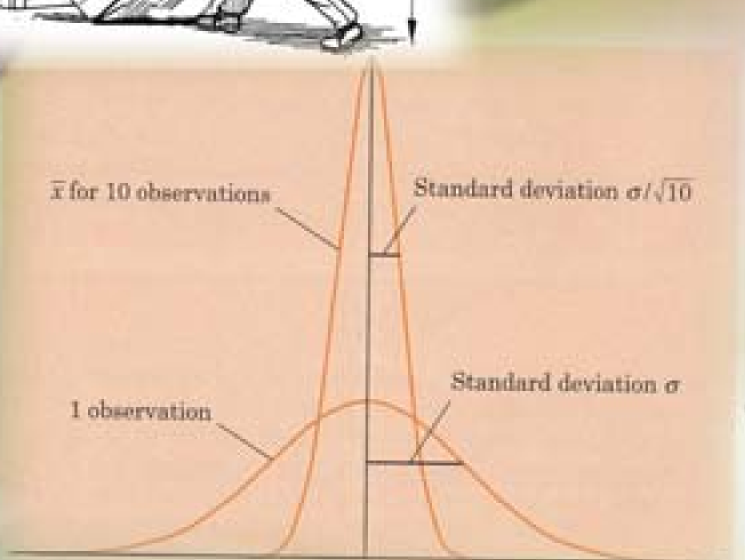
G. C. E. (Advanced Level)

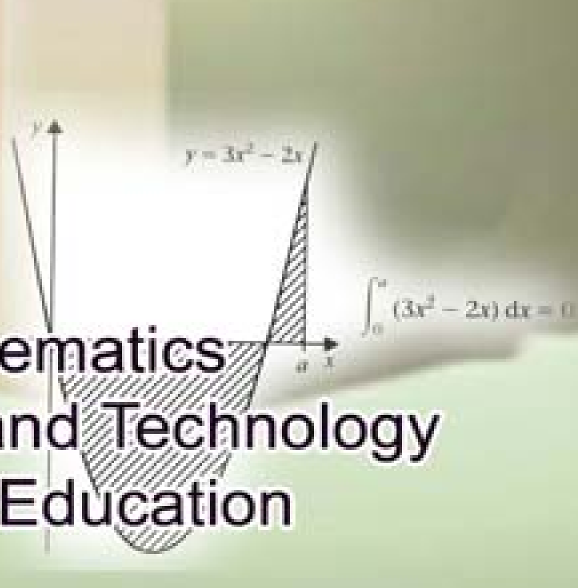
Mathematics

Grade 13

Teacher's Instructional Manual

(To be implemented from 2010)





Department of Mathematics
Faculty of Science and Technology
National Institute of Education
Sri Lanka

Mathematics

Grade 13

Teacher's Instructional Manual

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Department of Mathematics
Faculty of Science and Technology
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Maharagama
Sri Lanka

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Teacher's Instructional Manual

Grade 13 – 2010

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Content

Term I

Term I - Mathematics I

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
5.1	1. Explains the fundamental principle of counting	**Permutation and Combinations** **Fundamental principle of counting:** If one operation can be performed in m different ways and a second operation can be performed in n different ways for all ways of the operation one, then there will be $m \times n$ different ways performing the two operations in succession. Illustrate with examples.	12
	2. Defines factorial	Definition of factorial n; where n is a non-negative integer. Normal form : $0! = 1$ $n! = 1.2.3....n$, for $n \geq 1$ Recursive form : $F(0) = 1$ $F(n) = n\,F(n-1)$	
	3. Defines ${}^{n}p_{n}$ and obtain the formula for ${}^{n}p_{n}$.	Define that the number of permutation of n different objects taken all at a time is ${}^{n}p_{n}$ and obtain that ${}^{n}p_{n} = n!$ Here, n is a positive integer	
	4. Defines ${}^{n}p_{r}$ and finds formula for ${}^{n}p_{r}$	Define that the number of permutation of n different objects taken r $(o \leq r \leq n)$ at a time is ${}^{n}p_{r}$ and obtain ${}^{n}p_{r} = \dfrac{n!}{(n-r)!}$	
	5. Finds the permutations in which the objects may be repeated.	Show that the number of permutation of n different objects taken r $(o \leq r \leq n)$ at a time when each object may occur any number of time is n^{r}	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	6. Finds the permutations of n objects not all different.	Show that the number of permutation of n objects r of which are one kind and the remaining all are different is $\frac{n!}{r!}$	
	7. Explains the cyclic permutations.	Show that the number of permutation in which n different objects can be arranged round a circle is $(n-1)!$; where $n \geq 1$	
5.2	1. Defines combination.	Define that the number of combination of n different objects taken r at a time is ${}^nC_r$ and obtain $${}^nC_r = \frac{n!}{(n-r)!r!}$$ Show that (i) ${}^nC_r = {}^nC_{n-r}$ (ii) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$	15
	2. Explains the distinction between permutations and combinations.	Explain (with examples) that in permutation, the order is important, but in combination order is immaterial (neglected). Show that the total number of combinations of n different objects taken any number at a time is $2^n - 1$ Guide students to solve problems on permutations and combination.	
13.6	1. Identifies increasing functions and decreasing functions.	**Calculus** **Defining an increasing function:** Let f be a function defined on $(a,\ b)$ (i) If for every $x_1\ x_2 \in (a,b)$ $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$ then f is said to be monotonically increasing function on (a,b)	06

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		(ii) If for every $x_1\ x_2 \in (a,b)$ $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$ then f is said to be strictly increasing function on (a,b) **Defining a decreasing function** (i) If for every $x_1 , x_2 \in (a,b)$ $x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$ then f is said to be monotonically decreasing function on (a,b) (ii) If for every $x_1 , x_2 \in (a,b)$ $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$ then f is said to be strictly decreasing function on (a,b) **Note:** A constant function is said to be monotonic)	
	2. Explains increasing function and decreasing function using derivatives.	Let f be a differentiable function on (a,b). For all $x \in (a,b)$, if $f'(x) > 0$, then f is an increasing function on (a,b). For all $x \in (a,b)$, if $f'(x) < 0$, then f is decreasing function on (a,b).	
	3. Explains stationary points.	Let f be a function defined on (a,b). If there exists a point c (a,b) such that $f'(c) = 0$, then f has a stationary point at $x = c$. $f(c)$ is the stationary value of f.	
	4. Defines the local maximum/minimum value of a function.	(1) A function f is defined in a neighbourhood of a stationary point $x = a$ of f. If there exists $\delta > 0$ such that $f(x) < f(a)$ for all $x \in (a - \delta, a + \delta) - \{a\}$ then f has a local maximum at $x = a$.	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		(2) A function f is defined in a neighbourhood of a stationary point $x = a$ of f. If there exists $\delta > 0$ such that $f(x) > f(a)$ for all $x \in (a-\delta, a+\delta) - \{a\}$ then f has a local minimum at $x = a$.	
	5. Explains local maximum/minimum value of a function using derivatives.	Let f be a function, differentiable in the neighbourhood of a. If (i) $f'(a) = 0$ (ii) $f'(x) > 0$ for all $x \in (a-\delta, a)$ and (iii) $f'(x) < 0$ for all $x \in (a, a+\delta)$ then f has a local maximum at $x = a$. if (i) $f'(a) = 0$ (ii) $f'(x) < 0$ for all $x \in (a-\delta, a)$ and (iii) $f'(x) > 0$ for all $x \in (a, a+\delta)$ then f has a local minimum at $x = a$.	
	6. Defines points of inflexion of a function.	Let f be a differentiable function, in the neighbourhood of a. If (i) $f'(a) = 0$ (ii) there exist $\delta > 0$ such that for all $x \in (a-\delta, a+\delta) - \{a\}$ $f'(x) > 0$ or for all $x \in (a-\delta, a+\delta) - \{a\}$ $f'(x) < 0$ then f has a point of inflexion at $x = a$.	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	7. Uses the second derivative to test local maximum/minimum.	State that (i) If $f'(a) = 0$ and $f''(a) > 0$ then f has a local minimum at $x = a$. (ii) If $f'(a) = 0$ and $f''(a) < 0$ then f has a local maximum at $x = a$.	
	8. Uses derivatives to solve day to day problems.	Discuss the ways to solve problems involving local maximum and local minimum in day to day activities.	
13.7	Sketches the graphs of function.	Direct the students to sketch graphs of function using the above principles. Examples involving horizontal and vertical asymptotes are also included.	08
13.8	1.Defines integration as the reverse process of differentiation.	For a given function *f(x)*, if there exists a function F(x) such that $\frac{d}{dx}\{F(x)\} = f(x)$, then $F(x)$ is said to be the antiderivative of $f(x)$. The process is also called anti-differentiation.	02
	2. Explains the arbitary constant.	If $\frac{d}{dx}\{F(x) + C\} = f(x)$ then we write $\int f(x)dx = F(x) + C$ Where C is an arbitary constant. Discuss that integral of a function is not unique but can differ by a constant which is called an arbitary constant. The above form is an indefinite integral. **Note:** When solve problems students need to describe C. i.e., need to write that C is an arbitrary constant / constant of integration.	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
13.9	3. States the basic theorems of integration. 1. Identifies the indefinite integrals of the standard function.	Explain the following theorems i) $\int \{f(x) + g(x)\}dx = \int f(x)dx + \int g(x)dx$ ii) $\int \lambda f(x)dx = \lambda \int f(x)dx$ where *f(x)* and *g(x)* are functions of *x* and λ is a constant. State the followings: 1. (a) $\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ (b) $\int \frac{1}{x} dx = \ln\|x\| + C \quad (x \neq 0)$ (c) $\int e^x dx = e^x + C$ 2. $\int \sin x\, dx = -\cos x + C$ 3. $\int \cos x\, dx = \sin x + C$ 4. $\int \sec^2 x\, dx = \tan x + C$ 5. $\int \operatorname{cosec}^2 x\, dx = -\cot x + C$ 6. $\int \sec x \tan x\, dx = \sec x + C$ 7. $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$ Suppose anti-derivative of $f(x)$ is $g(x)$ then $\frac{d}{dx} g(x) = f(x)$. Explain $px + q\,(p \neq 0)$ substituted for *x* in $g(x)$, and differentiated with respect to *x*, $\frac{d}{dx}\left(\frac{1}{p} g(px+q)\right) = \frac{d}{d(px+q)} g(px+q)$ $= f(px+q)$ $\Rightarrow \int f(px+q)dx = \frac{1}{p} g(px+q) + C$	07

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	2. Integrates rational functions when the numerator is the derivative of the denominator.	$\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert + C$ where $f'(x)$ is the derivative of *f(x)*.	
	3. Integrates rational functions using partial fractions.	$\int \frac{P(x)}{Q(x)}dx$ where *Q(x)* is a polynomial of degree ≤ 4 and factorisable.	
	4. Integrates the trigonometric functions.	Using trigonometry and standard integrals obtain the following integrals. $\int \tan x dx,\ \int \cot x dx,\ \int \sec x dx.$ $\int \mathrm{cosec}\ x dx,\ \int \sin^2 x dx,\ \int \cos^2 x dx.$ $\int \sin mx \cos nx\, dx, \int \cos mx \cos nx\, dx$ $\int \sin mx \sin nx\, dx$	
13.10	Determines definite integral by using fundamental theorem of calculas.	Define $\int_a^b f(x)dx = [\phi(x)]_a^b = \phi(b) - \phi(a)$ where $\phi(x)$ is the integral of $f(x)$ and use it to evaluate model problems leading to integrals of all the standard forms discussed. Discuss the following theorems. (i) $\int_a^b \{f(x) + g(x)\}dx = \int_a^b f(x) + \int_a^b g(x)dx$ (ii) $\int_a^b \lambda f(x)dx = \lambda\int_a^b f(x)dx$ (iii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$	06

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
13.11	Use diverse methods for integration.	(iv) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ if and only if *f(x)* is integrable in [*a,c*] and [*c,b*] Discuss, substitute $t = f(x)$ $\int f'(x)\{f(x)\}^r dx = \int t^r dt$ $= \begin{cases} \frac{1}{r+1} t^{r+1}, \text{when } r \neq -1 \\ \ln\lvert t\rvert, \quad \text{when } r = 1 \end{cases}$ Also discuss the following integrals. (i) $\int \sin^m x\, dx$ (ii) $\int \cos^m x\, dx$ (iii) $\int \sin^m x \cos^n x\, dx$ where *m* , *n* are positive integers (iv) $\int \sqrt{a^2 - x^2}\, dx$	06

Term I - Mathematics II

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
5.1	1. Explains random experiment	**Probability** Discuss what is random experiment. Give some examples for random experiments.	05
	2. Defines sample space.	The set of all possible outcomes for an experiment is called the sample space for that experiment.	
	3. Defines an event.	An event is a subset (proper or non proper) of a sample space. i.e. An event is a collection of one or more of the outcomes of an experiment.	
	4. Explains event space.	Set of all events of a random experiment is said to be an event space. Note that the null set and the sample space itself are also members of the event space.	
	5. Explains simple events and compound events.	An event that includes one and only one of the outcomes of an experiment is called a simple event. A compound event is a collection of more than one outcome of an experiment Explain (i) Union of two events (ii) Intersection of two events. (iii) Mutually exclusive events. (iv) Collectively exhaustive events. (v) Complementary event of an event.	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
5.2	1. States classical definition of probability.	The probability of an event "A" related to a random experiment consisting of N equally probable events is defined as $P(A) = \frac{n(A)}{N}$ Where $n(A)$ is the number of simple events in the event A. **Limitations** (i) The above formulae cannot be used when the results of the random experiment are not equally probable. (ii) When the sample space is infinite the above formulae is not valid.	10
	2. States the experimental definition of probability.	The probability of an event is calculated from the results of the experiment after the series of trials has been completed. If the event A occurs N_A times in N trials, then the fraction $\frac{N_A}{N}$ tend to a limit, called the probability of A, as N tends to infinity. i.e. $P(A) = \lim_{N \to \infty} \frac{N_A}{N}$ Note that this is also known as the relative frequency approach to probability.	
	3. States the axiomotic definition.	Let $\mathcal{S}$ be the event space corresponding to a sample space Ω of a random experiment. A function $P : \mathcal{S} \longrightarrow [0,1]$ satisfying the following conditions: (i) $P(A) \geq 0$ for any $A \in \mathcal{S}$ (ii) $P(\Omega) = 1$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		(iii) If A_1 and A_2 are mutually exclusive events, then $P(A_1 \cup A_2) = P(A_1) + P(A_2)$ is said to be a probability function. Note that axiomatic definition cannot be used to find the probability of an event but it can be used to find the probability of complex events when probabilities are given.	
	4. Proves the theorems on probability using axiomatic definition and solves problems using the above theorems.	Prove that (i) $P(\phi) = 0$ (ii) $P(A') = 1 - P(A)$ (iii) $P(A) = P(A \cap B) + P(A \cap B')$ (iv) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (v) If $A \subseteq B$, then $P(A) \leq P(B)$ Where A, B are events in an experiment and A' represents the compliment of A.	
5.3	1. Defines conditional probability.	Let Ω be the sample space of a random experiment and A and B be two events where P(A) >0, then the conditional probability of the event B given that the event A has occured, denoted P(B/A) is defined as $$P(B/A) = \frac{P(A \cap B)}{P(A)}$$	07
	2. Proves the theorems on conditional probability.	Prove that (i) If P(A) >0 then $P(\phi/A) = 0$ (ii) If A, $B \in \mathcal{S}$ and P(A) >0 then $P(B'/A) = 1 - P(B/A)$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		(iii) If $A, B_1, B_2 \in \mathcal{E}$ and $P(A) > 0$ then $P(B_1 \mid A) = P(B_1 \cap B_2/A) + P(B_1 \cap B_2'/A)$ and $P(B_1 \cup B_2/A) = P(B_1/A) + P(B_2/A) - P[B_1 \cap B_2/A]$	
	3. States multiplication rule.	Let A_1, A_2 be any two events in an experiment and $P(A_1) > 0$ $P(A_1 \cap A_2) = P(A_1) \cdot P(A_2/A_1)$ State multiplication rule for three events. i.e; $P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2/A_1) \cdot P(A_3 / A_1 \cap A_2)$	
5.4	1. Defines independent events.	Let A_1, A_2 be two events in the event space A_1 and A_2 are said to be independent if and only if $P(A_1 \cap A_2) = P(A_1) \cdot P(A_2)$	07
	2. Proves theorems on independent events and applies to solve problems.	If A and B are independent events then (i) A and B' (ii) A' and B (iii) A' and B' are independent.	
	3. Explains independence of three events.	Let A, B, C be three events in the event space corresponding to the sample space Ω of a random experiment. If (i) $P(A \cap B) = P(A) \cdot P(B)$ (ii) $P(B \cap C) = P(B) \cdot P(C)$ (iii) $P(A \cap C) = P(A) \cdot P(C)$ (iv) If $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$ then A, B and C are said to be independent of each other.	

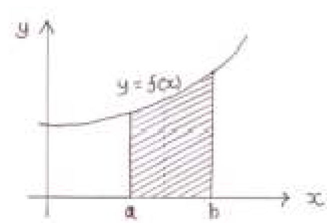
Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
5.5	1. Defines partition of a sample space	Let $B_1, B_2, B_3, ...B_n$ be a sequence of events in the event space corresponding to the sample space Ω of a random experiment. If (i) $B_i \cap B_j = \phi$ for all $i \neq j$ and (ii) $\bigcup_{i=1}^{n} B_i = \Omega$ then $\{B_1, B_2, ...B_n\}$ is called a partition of the sample apace Ω.	06
	2. States the theorem on total probability and applies to solve problems.	Let $\{B_1, B_2, ...B_n\}$ be a partition of the event space $\mathcal{S}$ corresponding to the sample space Ω of a random experiment. If $P(B_i) > 0$ and A is any event in the event space. then $$P(A) = \sum_{i=1}^{n} P(A/B_i) \cdot P(B_i)$$	
	3. States Bayes theorem and applies to solve problems.	Let $B_1, B_2, ...B_n$ be a partition of the event space corresponding to the sample space Ω of a random experiment. If A is any event in $\mathcal{S}$ and $P(A) > 0$ then $$P(B_j \mid A) = \frac{P(A \mid B_j)) P(B_j)}{\sum_{i=1}^{n} P(A \mid B_i) \cdot P(B_i)}$$	

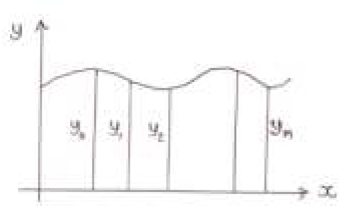
Term II

Term 2 - Mathematics I

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
6	1. Explains Pascal triangle 2. States binomial theorem for positive integral index.	**Binomial Expansion** 1 1 2 1 1 3 3 1 1 4 6 4 1 This array of numbers which is such that each number , except those at the ends, is the sum of the two numbers on the either side of it in the line above, known as Pascal triangle. Explain that $(1+x)^1 = 1+x$ $= {}^1C_0 + {}^1C_1.x$ $(1+x)^2 = 1+2x+x^2$ $= {}^2C_0 + {}^2C_1.x + {}^2C_2x^2$ $(1+x)^3 = 1+3x+3x^2+x^3$ $= {}^3C_0 + {}^3C_1.x + {}^3C_2x^2 + {}^3C_3x^3$ Discuss $(1+x)^4$ and $(1+x)^5$ Statement of the theorem for positive integral index. $(a+x)^n = {}^nC_0a^n + {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2$ $+ \ldots\ldots\ldots\ldots + {}^nC_nx^n$ $= \sum_{r=0}^{n} {}^nC_ra^{n-r}x^r$ where ${}^nC_r = \frac{n!}{(n-r)!r!}$ $(0 \le r \le n)$ In the expansion (i) ${}^nC_0, {}^nC_1, {}^nC_2 \ldots\ldots {}^nC_n$ are called binomial coefficients.	12

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		(ii) ${}^nC_0a^n, {}^nC_1a^{n-1}, \ldots {}^nC_n$ are called coefficients of the expansion. (iii) The number of terms in the expansion is $n+1$ (iv) General term T_{r+1} is given by . $T_{r+1} = {}^nC_r a^{n-r}.x^r$ Note that the powers of x are in ascending order .	
	3. Uses binomial theorem to solve problems.	Obtain the expansion of $(1 + x)^n$ Simple applications using binomial expansion.	
		Inequalities	
4.2	1. States the modulus (absolute value) of a real number .	Let $x \in \mathbb{R}$ Define $\|x\| = x$, if $x \geq 0$ $= -x$, if $x < 0$	08
	2. Defines the modulus function.	Let $f : \mathbb{R} \to \mathbb{R}$ be a function $\|f\|$ is defined as follows: $\|f\| : \mathbb{R} \to \mathbb{R}$ $\|f\|(x) = \|f(x)\|$ i.e $\|f\|(x) = f(x)$, if $f(x) \geq 0$ $= -f(x)$, if $f(x) < 0$ Illustrate with examples.	
	3. Draws the graphs of functions including modulus.	Graphs of the functions such as $y = \|ax\|$, $y = \|x - a\|$, $y = \|ax\| + b$ $y = \|ax + b\| + c$ $y = c - \|ax + b\|$ $y = \|ax + b\| \pm \|cx + d\|$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		$y = \left\|ax^2 + b + c\right\|$ where $a,b,c,d \in \mathbb{R}$	
	4. Solves inequalities involving modulus.	Determination of solution set of inequalities such as $\left\|ax+b\right\| \gtreqless \left\|cx+d\right\|$ $\left\|ax+b\right\| \gtreqless cx+d$ $\left\|x+a\right\| + \left\|x+b\right\| \gtreqless \left\|x+c\right\|$ (i) algebraically (ii) graphically	
		Calculus	
13.12	Integrates using the method of integration by parts.	Let $u(x)$ and $v(x)$ be differentiable functions and show that $\int\left(u\frac{dv}{dx}\right)dx = uv - \int\left(v\frac{du}{dx}\right)dx$ Discuss problems by using integration by parts.	06
13.13	1. Finds the area under a curve.	Define the area under the curve as a definite integral. Let $y = f(x)$ be a continuous function, provided $f(x) \geq 0$ for $x \in [a,b]$  In general, area bounded by the curve $y = f(x)$ and x axis and the lines $x = a$ and $x = b$ is given by $\int_a^b f(x)dx$	04

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
13.14	2. Finds the area between two curves. Uses the methods of approximation to solve problems.	This is referred to as an area under the curve $y = f(x)$ from $x = a$ and $x = b$. Let $y = f(x)$, $y = g(x)$ be the two curves such that $f(x) \geq g(x)$ in the interval $[a, b]$. The area bounded by the two curves and the lines $x = a$, $x = b$ is given by $$\left\lvert \int_a^b \{f(x) - g(x) dx\} \right\rvert$$ In general $\int_a^b \lvert f(x) - g(x) \rvert dx$ Discuss the following approximation methods for evaluating a definite integral. **(i). The trapezium rule:**  Let the area represented by $\int_a^b f(x)dx$ be divided into equal strips of width h $$\int_a^b f(x)dx = \frac{1}{2}h\left(y_0 + y_1\right) + \frac{h}{2}\left(y_1 + y_2\right)$$ $$\ldots\ldots\ldots\ldots + \frac{h}{2}\left(y_{n-1} + y_n\right)$$ $$= \frac{h}{2}\left[\left(y_0 + y_n\right) + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$	04

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
7.1	1. Defines a sequence.	where $h = \dfrac{b-a}{n}$ **2. Simpson's Rule** Suppose that the area represented by $\int_a^b f(x)dx$ is divided into 2n strips each of width h. Simpson's rule is given by, $\int_a^b f(x)dx \approx \frac{h}{3}[(y_0 + y_{2n})$ $+4(y_1 + y_3 + .. + y_{2n-1})$ $+2(y_2 + y_4 + .. + y_{2n-2})]$ Note that Simpson's rule requires even number of strips (or odd number of ordi-nates) **Series** Definition of a sequence as a set of terms in a specific order with a rule for obtaining terms. If a_n is the nth term of a sequence, the sequence is denoted by $\{a_n\}$ $\{a_n\}$ is said to be convergent if $\lim_{n\to\infty} a_n$ exists (finite number) Otherwise the sequence is said to be divergent.	05

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
7.4	Interprets the limit of a sequence.	(1) Discuss the following limits: $\lim_{n\to\infty}\left(\frac{1}{n}\right)$ $\lim_{n\to\infty}\left(\frac{1}{n^2}\right)$ $\lim_{n\to\infty}\left(\frac{1}{2^n}\right)$, $\lim_{n\to\infty}\left(\frac{1}{r^n}\right)$ $\lim_{n\to\infty}\left(\frac{an+b}{cn+d}\right)$ $\lim_{n\to\infty}\left(\frac{an+b}{pn^2+qn+r}\right)$ $\lim_{n\to\infty}\left(\frac{an^2+bn+c}{pn+q}\right)$ (2) Discuss the limit of a sequence	05
7.1	2. Defines a series.	Connection between a sequence and series. Partial sum of a sequence terms is called a series. Example : $S_n=\sum_{r=1}^{n}a_r$	
	3. States fundamental theorems on summation.	State the general term of a series is Ur and The sum of n terms as, $\sum_{r=1}^{n}U_r$, $n=1,2,3,\ldots$ Show that (i) $\sum_{r=1}^{n}(U_r+V_r)=\sum_{r=1}^{n}U_r+\sum_{r=1}^{n}V_r$ (ii) $\sum_{r=1}^{n}kU_r=k\sum_{r=1}^{n}U_r$ where k is a constant. In general $\sum_{r=1}U_rV_r\neq\left(\sum_{r=1}^{n}U_r\right)\left(\sum_{r=1}^{n}V_r\right)$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	4. Finds the sum of an arithmetic series.	Definition of an arithmetic series A series, which after the first term, the difference between a term and the preceding term is constant, is called an Arithmetic series or Arithmetic Progression. (1) Show that the general term T_r $T_r = a + (r - 1)\ d$, where a is the first term and d is the common difference and (2) The sum of n terms $S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}\left[a + l\right]$ where l is the last erm of the series. Application of the above formulae.	
	5. Finds the sum of a geometric series.	Definition of a geometric series A series which after the first term, the ratio between a term and the preceeding term is constant, is called geometric series. (i) Show that the general term $T_n = ar^{n-1}$ where a is the first term and r is the common ratio (ii) Show that the sum of n terms S_n, $S_n = \frac{a\left(1 - r^n\right)}{1 - r} \quad (r \neq 1)$ $= na \quad (r = 1)$ Application of the above formulae.	
7.2	Finds the sum of the series.	(1) Determination of $\sum_{r=1}^{n} r,\ \sum_{r=1}^{n} r^2\ \sum_{r=1}^{n} r^3$ and the use of the above results and the use of fundamental theorems on summation. (2) Find the summation of series using (i) Method of difference (ii) Method of partial fractions.	08

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
7.3	Uses the principle of Mathematical induction.	Explain the proof of Mathematical Induction. Use of the principle of Mathematical Induction in proving results such as (i) $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$ (ii) $\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(2n+1)}{3}$ (iii) $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$ (iv) $\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$	05
7.4	2. Analyses the sum of terms to infinity.	Let $\sum U_r$ be a series and $S_n = \sum_{r=1}^{n} U_r$ If $\lim_{n \to \infty} S_n = l$ (finite), then the series $\sum_{r=1}^{\infty} U_r$ is said to be convergent and the sum to infinity is l i.e. $\sum_{n=1}^{\infty} U_n = l$ Otherwise the series is said to be divergent. In an infinite geometric series with the first term a and common ratio r, the series is convergent if $\lvert r \rvert < 1$ and the sum to infinity is $\frac{a}{1-r}$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	3. Explains difference equations	Identify a difference equation as a sequence $\{x_n\}_{n=0}^{\infty}$ for which the n^{th} term $x_n = f(n)$ for $n \geq 1$ and an initial condition/initial conditions is / are known. **Example 1** Population x_t of a species after t number of years. Suppose they are growing at a rate of 2% per year with initial population of x_0. Difference equation $x_{t+1} = x_t + \frac{2}{100}x_t$ with x_0 is known. **Example 2** Radium decays at the rate of 1% every 25 years. After 25n number of years let the amount of the radium be x_n. then $x_{n+1} = x_n - \frac{1}{100}x_n$ and x_0 is known. **Example 3** Compound interest Initial amount of the investment is P, rate of interest is r. x_t - amount of the investment after t number of years, then $x_{t+1} = x_t + rx_t$ and $x_0 = P$	05

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	Classifies difference equations.	A difference equation of the form $x_{n+1} = ax_n + b$ where a and b are real valued constants and $a \neq 0$ is called 1st order linear difference equation. If $b = 0$, then it is called a homogeneous difference equation.	
	Obtains the solution of difference equation.	**Solution of** $x_n = ax_{n-1} + b$ $x_n = ax_{n-1} + b$ $= a[ax_{n-2} + b] + b$ $= a^2 x_{n-2} + b(1 + a)$ $\therefore x_n = a^2[ax_{n-3} + b] + b(1 + a)$ $= a^3 x_{n-3} + b(1 + a + a^2)$ Following this procedure we get $x_n = a^n x_0 + b(1 + a + a^2 + ..a^{n-1})$ Now consider two cases if $a = 1$ $x_n = x_0 + nb$ Otherwise $x_n = a^n x_0 + \frac{1 - a^n}{1 - a} b$. For the homogeneous equation, Solution $x_n = x_0$ when $a = 1$	

Term 2 - Mathematics II

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
5.6	1. Explains random variables.	**Statistics** Let Ω be the sample space of a random experiment. A random variable is a function from sample space Ω to set of real numbers real line and is denoted by X, Y, Z .. etc. $X : \Omega \rightarrow \mathbb{R}$ is a function $X(\omega) = x, \ w \in \Omega, \ x \in \mathbb{R}$	02
	2. Defines discrete random variable.	Let X be a random variable. ie, $X : \Omega \rightarrow \mathbb{R}$ is a function If the set of values of X. (range of X) is finite or countably finite, then the random variable is said to be discrete.	
	3. Defines continuous random variable.	Let X be a random variable ie, $X : \Omega \rightarrow \mathbb{R}$ is a function If the values of X has one or more than one interval X is said to be continuous random variable.	
5.7	1. Defines the probability mass function for a discrete random variable.	Let Ω be the sample space of a random experiment and X be the random variable defined on Ω $X : \Omega \rightarrow \mathbb{R}$ Let the values of X be $\{ x_1, \ x_2, x_3, \ldots x_n \}$ A function p is defined on $\{ x_1, \ x_2, \ldots x_n \}$ as follows $P(X = x)$ means probability of $X = x$. ie. $p(x) = \begin{cases} P(X = x), & x = x_i, i = 1, 2, \ldots n \\ 0 & \text{Otherwise} \end{cases}$	06

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		p(x) is said to be probability mass function of X. The set of ordered pairs $\{(x_i, p(x_i)) : i = 1, 2, ...n\}$ is the probability mass function. It can be shown in a table as follows: <table><tr><td>x</td><td>x_1</td><td>x_2</td><td></td><td>x_n</td></tr><tr><td>$p(x)$</td><td>$p(x_1)$</td><td>$p(x_2)$</td><td></td><td>$p(x_n)$</td></tr></table>Properties of $p(x)$ (i) $p(x_i) \geq 0 \quad (i = 1, 2, ...n)$ (ii) $\sum_{i=1}^{n} p(x_i) = 1$	
	2. Explains the probability density function for a continuous random variable.	The probability density function (*p.d.f*) corresponds to a "smoothed out" relative frequency histogram for which the area under the curve equals the probability. Hence the total area must be one. **Properties of $f(x)$** (i) $f(x) \geq 0$ for all x and (ii) $\int_{-\infty}^{+\infty} f(x)dx = 1$ (iii) $\left[P(a < X < b) = \int_{a}^{b} f(x)dx \right]$	
5.8	1. Defines mathematical expectation, variance and standard deviation of a discrete random variable.	Let $p(x)$ be the probability mass function corresponding to a discrete random variable X. $p(x) = \begin{cases} P(X = x), & x = x_i, i = 1, 2, ..n \\ 0 & \text{Otherwise} \end{cases}$	05

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		Mean of X or Expected value of X, denoted E(x) and $E(x) = \sum_{i=1}^{n} x_i p(x_i)$ Variance of X, denoted Var(x) and $\text{Var}(x) = E[X - E(x)]^2$ Show that $E[X - E(x)]^2 = E(X^2) - [E(x)]^2$ $\left(E(x^2) = \sum_{i=1}^{n} x_i^2 p(x_i)\right)$ Standard deviation denoted and $\sigma = \sqrt{\text{Var}(X)}$ If a, b are constants, show that $E(aX + b) = aE(x) + b$ and $\text{Var}(ax + b) = a^2 \text{Var}(X)$	
	2. Defines expected value and variance of a continuous random variable X.	Let $f(x)$ be a probability density function for a continuous random variable X. Mean of X or Expected value of X, denoted E (x) and $E(X) = \int_{-\infty}^{+\infty} x \cdot f(x) dx$ Variance of X, denoted Var(x) and $\text{Var}(x) = E[X - E(x)]^2$ Show that $E[X - E(x)]^2$ $= E(X^2) - [E(X)]^2$ $\left(E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x) dx\right)$ Standard deviation of X $= \sqrt{\text{Var}(x)}$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
5.9	Explains cumulative distribution function of a random variablnle X.	In a probability distribution the probabilities up to a certain value of x are summed to give a cumulative probability. The cumulative probability function is written as $F(x)$. For a discrete random variable X with probability mass function p (X) $p(X) = \begin{cases} P(X = x), & x = x_1, x_2, \ldots x_n \\ 0 & \text{Otherwise} \end{cases}$ the cumulative distribution function is given by $F(t)$ $F(t) = P(X \leq t)$ where $= \sum_{x=x_1}^{t} P(X = x_i)$ For a continuous random variable X with probability density function $f(x)$, the cumulative distribution function is given by $F(t)$ Where $F(t) = P(X \leq t)$ $= \int_{-\infty}^{t} f(x)dx$	02
		Linear programming	
6.1	1. Explains what is linear programming.	Linear programming is a mathematical optimization technique. i.e, a method attempts to maximize or minimize a particular objective under certain constraints. Example : Maximize profit Minimize cost	12

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	2. States the types of problem.	Discuss the following types. (i) No answer problems (ii) Single answer problems (iii) Multiple answer problems.	
	3. Constructs linear programming models.	Explain the following terms (with examples) in the formulation of the linear programming model. Decision variable Objective function Constraints Non-negative conditions. Discuss various linear programming models. ***Example*** Min or Max of $Z = ax + by$ Subject to $cx + dy \leq k_1$ $ex + fy \geq k_2$ $x \geq 0, y \geq 0$ Construct linear programming models with more than two variables.	
6.2	1. Describes the graphical method of solving linear programming problems.	Explain graphical method of solving linear programming models with two decission variables. Use suitable examples.	06
	2. Identifies feasible region.	Explain (i) feasible solutions of linear programming (ii) feasible region of linear programming	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		Discuss the solution of (i) Maximizing model Example : Profit (2) Minimizing model Example : Cost	
	3. Identifies optimal solutions.	Explain the optimal solution if exists in a linear programming model. Discuss these possibilities (i) No solutions (ii) One unique solution (iii) Infinite number of solutions.	
		Note: Explain that models with more than two variables can be solved by using the method called simplex method. With the development of computers, solution procedures have become simple. MS, Excel can be used to solve the problems. No need to discuss the solution procedure. Objective is to let students that there are other methods to solve problems with more than two variables.	

Term III

Term 3 - Mathematics I

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
9.1	1. Defines a matrix	**Matrices** Matrix is a rectangular array of numbers. Matrices are denoted by alphabets A, B, C, . . . $A = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} \cdots & a_{mn} \end{bmatrix}$	05
	2. States the order of a matrix.	A matrix A has m rows and n coloumns. The size (order) of the matrix A is $m \times n$. A can be written as $(a_{ij})_{m \times n}$. Element of a matrix: a_{ij} is the element of matrix A in the i th row and j th coloumn. Row matrix: A matrix which has only one row is called a row matrix or row vector. Coloumn matrix: A matrix which has only one coloumn is called a coloumn matrix or coloumn vector. Null matrix : A matrix with every element is zero, is called null matrix.	
	3. Defines the equality of matrices.	Let A and B be two matrices of same order. $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$. If $a_{ij} = b_{ij}$ for all i, j then A = B.	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	4. Defines the addition of matrices.	State the condition for two maxtrices to be added. Maxtices are in the same order. Then corresponding elements are added. Let A = $(a_{ij})_{m\times n}$, B = $(b_{ij})_{m\times n}$ Then A + B = $(a_{ij})_{m\times n}$ + $(b_{ij})_{m\times n}$ = $(a_{ij}+b_{ij})_{m\times n}$ Note that (i) Addition is closed. (ii) Addition is comulative A + B = B + A Addition is associative. (A + B) + C = A + (B + C)	
	5. Defines the multipli-cation of a matrix by a scalar.	Let A = $(a_{ij})_{m\times n}$ and $\lambda\in\mathbb{R}$ $\lambda A = (\lambda a_{ij})_{m\times n}$. for all i, j When $\lambda = -1$ (–1)A = – A is called the negative of the matrix A. Let A, B be two matrices of same order. Then, A – B = A + (–1) B.	
	6. Defines the multipli-cation of matrices.	Let A = $(a_{ij})_{m\times p}$ and B = $(b_{ij})_{q\times n}$ When $p = q$, the product AB is defined. If A = $(a_{ij})_{m\times p}$ and B = $(b_{ij})_{p\times n}$ then AB = $\left(\sum_{k=1}^{p}(a_{ik}b_{kj})\right)_{m\times n}$ is order of $m \times n$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		Discuss that (i) Even AB is defined, BA is not necessarily defined. (ii) In general AB $\neq$ BA.	
9.2	1. Explains special cases of matrices.	In a matrix A of order $m \times n$ when $m = n$ A is defined as square matrix of order n Let A be a square matrix of order n. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ & & & & \\ a_{n1} & a_{n2} & \cdots & & a_{nn} \end{bmatrix}$ $(a_{11}, a_{22}, a_{33}, \ldots a_{nn})$ is the leading (principal) diagonal. * A square matrix A of order n is said to be identity matrix if $a_{ij} = 1$ when $i = j$ 0 when $i \neq j$ and denoted by I_n. * A square matrix A is said to be diagonal if $a_{ij} = 0$ for all $i \neq j$	07
	2. Uses theorems in solving problems.	For square matrices A, B and C. A(BC) = (AB) C (Associative) under multiplication. A(B+C) = AB + AC (distributive) (B + C)A = BA + CA (distributive) A+ 0 = A = 0+ A [0 – square matrix] A $\times$ I= A =I $\times$ A	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		Note that AB = 0 does not necessarily follow that A = 0 or B = 0. [0 – zero matrix] When $f(x)$ is a polynomial in x computation of $f(A)$, where A is a square matrix.	
	3. Defines the transpose of a matrix.	Let A be a matrix of order $m \times n$. $A = (a_{ij})_{m \times n}$ Transpose of A, denoted A^T, is defined by $A^T = (b_{ij})_{n \times m}$ Where $b_{ij} = a_{ji}$ for all i, j. Properties of matrix transpose $(A + B)^T = A^T + B^T$ $(KA)^T = K.A^T$, $k \in \mathbb{R}$ $(A^T)^T = A$ $(AB)^T = B^T \cdot A^T$	
	4. Defines the minor of an element in a 3×3 matrix.	Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ be a 3×3 matrix. Then minor of an element in i th row and jth coloumn, denoted by M_{ij}, is a 2×2 determinant obtained by deleting i th row and j th coloumn of A, where $i, j = 1,2,3$. For example minor of a_{12} is $M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$ $= a_{21} \cdot a_{33} - a_{31} \cdot a_{23}$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	5. Defines the cofactor of an element in a 3×3 matrix.	Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ be a 3×3 matrix. Cofactor of the element $a_{ij} (1 \leq i, j \leq 3)$, denoted by A_{ij} is given by $A_{ij} = (-1)^{i+j} M_{ij}$	
9.3	1. Defines the inverse of a matrix.	Given a square matrix A, if there exists a matrix B such that AB = I = BA, then B is said to be the inverse of A and is denote by A^{-1} $AA^{-1} = I = A^{-1}A$. Notice that matrix inversion is defined for square matrices only. Properties of matrix inversion. $(A^{-1})^{-1} = A$ $(AB)^{-1} = B^{-1} A^{-1}$ $(A^{-1})^{T} = (A^{T})^{-1}$	05
	2. Finds the inverse of a 2×2 matrix.	Given a matrix $A = \begin{pmatrix} a, b \\ c, d \end{pmatrix}$, determinant of A = det (A) = $\lvert A \rvert$ = ad - bc. When $\lvert A \rvert \neq 0$, obtain $A^{-1} = \frac{1}{\lvert A \rvert}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$	
	3. Solve simaltaneous equations in two variable using matrices.	Let $a_1 x + b_1 y = c_1$ $a_2 x + b_2 y = c_2$ Writing the above equations in the form AX = C,	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
8.1	1. Expands determinants.	where A $= \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $C = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ When A^{-1} exists, $A^{-1}AX = A^{-1}C$ $X = A^{-1}C$ Discuss the solutions of simaltaneous equations. (i) unique solution. (ii) Infinite number of solutions. (iii) No solutions. **Determinants** (a) State the forms of 2×2 and 3×3 determinants. Expansion of a 2×2 determinant If $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ then $\Delta = a_1b_2 - a_2b_1$, where a_1, a_2, b_1, b_2 are real numbers. (b) Expansion of a 3×3 determinant Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then $\Delta = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$ $= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$	10

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	2. States the properties of determinants.	where $a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$ are real numbers. Note: We can expand determinant along a row or along a coloumn. We get the same result Discuss the following properties for 2×2 and 3×3 determinants. 1. If Δ_2 is obtained from Δ_1 by inter-changing two rows (coloumns) of Δ_1, then $\Delta_2 = -\Delta_1$ 2. If two rows (coloumns) of a determi-nant are equal, then determinant is zero. 3. The value of the determinant is unal-tered if a multiple of any row (coloumn) is added to any other row (coloumn). 4. If one row (coloumn) of a determinant (Δ) is multiplied by a scalar λ, the resulting determinant is equal to $\lambda\Delta$. 5. If all the elements in a row (or coloumn) are zero the value of determinant is zero. 6. Let $\Delta = \begin{vmatrix} x_1 & y_1 & a_1+b_1 \\ x_2 & y_2 & a_2+b_2 \\ x_3 & y_3 & a_3+b_3 \end{vmatrix}$ $\Delta_1 = \begin{vmatrix} x_1 & y_1 & a_1 \\ x_2 & y_2 & a_2 \\ x_3 & y_3 & a_3 \end{vmatrix}, \Delta_2 = \begin{vmatrix} x_1 & y_1 & b_1 \\ x_2 & y_2 & b_2 \\ x_3 & y_3 & b_3 \end{vmatrix}$ Then $\Delta = \Delta_1 + \Delta_2$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
8.2	1. Uses determinants to solve simalta- neous equations.	Discuss the solutions of two simaltaneous equations. $a_1x + b_1y + c_1 = 0$ ——— (i) $a_2x + b_2y + c_2 = 0$ ——— (ii) Use Cramer 's rule in solving equations. $\dfrac{x}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}} = \dfrac{-y}{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}} = \dfrac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$ Discuss the solutions of equations in three variables. $a_1x + b_1y + c_1z + d_1 = 0$ $a_2x + b_2y + c_2z + d_2 = 0$ $a_3x + b_3y + c_3z + d_3 = 0$ Use cramer 's rule in solving equations. $\dfrac{x}{\begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}} = \dfrac{-y}{\begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix}} = \dfrac{z}{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}$ $= \dfrac{-1}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$	06
12.1	1. Defines circle as a locus.	**Circles** Define a circle as the locus of a point which moves in a plane such that its distance from a fixed point is always constant. The fixed point is said to be the centre of the circle. The constant distance is the radius of the circle.	02

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	2. Obtains the equation of a circle.	Equation of the circle with centre (0,0) and radius r is $x^2 + y^2 = r^2$ Equation of the circle with centre (a, b) and radius r is $(x-a)^2 + (y-b)^2 = r^2$	
	3. Interprets the general equation of a circle.	General equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$. Obtain that the centre is $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$ $(g^2 + f^2 - c \geq 0)$	
	4. Finds the equation of the circle when the end points of a diameter are given.	Show that the equation of the circle with the points (x_1, y_1), (x_2, y_2) as the ends of a diameter is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$	
12.2	Identifies the position of a point with respect to a circle.	Given a point p = (x_0, y_0) and the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, explain that the point P lies inside the circle or on the circle or outside the circle according as $x_0^2 + y_0^2 + 2gx_0 + 2fy_0 + c \lesseqgtr 0$	01
12.3	1. Discusses the position of a straight line with respect to a circle.	Let $U \equiv lx + my + n = 0$ be a straight line and $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be a circle. By considering, (i) discriminant of the equadratic equation in x or y, obtained by solving S = 0 and U = 0 (ii) radius of the circle and the distance between the centre of the circle and the straight line.	04

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		Discuss whether (a) the line intersects the circle (b) the line touches the circle (c) the line lies outside the circle in both situation (i) and (ii)	
	2. Obtains the equation of the tangent at a point on the circle.	Show that the equation of the tangent at P$\left(x_0, y_0\right)$ on S is $xx_0 + yy_0 + g(x + x_0) + f(y + y_0) + c = 0$	
12.4	1. Finds the length of the tangents drawn to a circle from an external point.	Let S $\equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be a circle and P$\left(x_0, y_0\right)$ be an external point. Show that the length of the tangent is $\sqrt{x_0^2 + y_0^2 + 2gx_0 + 2fy_0 - c}$	05
	2. Finds the equations of the tangent drawn to a cicle from an external point.	Obtain the equations of tangents drawn to a circle from an external point.	
	3. Obtains the equation of the chord of contact of the tangents.	Let S $\equiv x^2 + y^2 + 2gx + 2fy + c = 0$ be a circle and P $= \left(x_0, y_0\right)$ be an external point. Show that the equation of chord of contact is $xx_0 + yy_0 + g(x + x_0) + f(y + y_0) + c = 0$	

Term 3 - Mathematics II

<table>
<tr><th>Competency Level</th><th>Learning Outcomes</th><th>Guidelines for subject matter</th><th>Number of periods</th></tr>
<tr>
<td>5.10</td>
<td>1. Explains Bernoull's distribution to calculate probability.</td>
<td>

Statistics

Let X be a random variable with probabilities $(1-\theta)$ and θ $(0<\theta<1)$ and take the values 0 and 1 respectively.

Then X follows a Bernoulll's distribution with parameter θ. The probability mass function $p(x)$ is given by

$p(x) = \theta^x(1-\theta)^{1-x}$ if $x = 0, 1$

$= 0$ otherwise.

The distribution is shown in the table as follows:

<table>
<tr><td>x</td><td>0</td><td>1</td></tr>
<tr><td>$p(x)$</td><td>$1-\theta$</td><td>θ</td></tr>
</table>

Note: The Bernulli distribution is the building block of creating distributions such as Binomial.

Illustrate with example.

Suppose that a bag contains 6 white balls and 3 red balls of same size. A ball is taken randomly from the bag. Let X be the random variable which represent the number of red balls.

Now $p(x) = \left(\frac{2}{3}\right)^x\left(1-\frac{2}{3}\right)^{1-x}$ if $x = 0,1$

$= 0$ otherwise.

<table>
<tr><td>x</td><td>0</td><td>1</td></tr>
<tr><td>$p(x)$</td><td>$\frac{1}{3}$</td><td>$\frac{2}{3}$</td></tr>
</table>

In a Bernoullis distribution obtain that

$E(X) = \theta$ and $Var(X) = \theta(1-\theta)$

</td>
<td>15</td>
</tr>
</table>

<table>
<tr><th>Competency Level</th><th>Learning Outcomes</th><th>Guidelines for subject matter</th><th>Number of periods</th></tr>
<tr><td></td><td>2. Explains discrete uniform distribution.

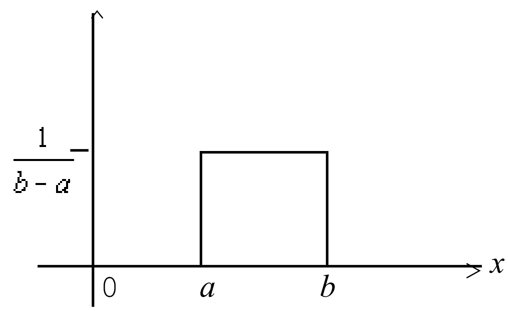
3. Explains Binomial distribution to calculate probabilities.</td><td>Let the random variable X be defined over the set of n distinct values $x_1, x_2, x_3, \ldots x_n$ which are all equally likely
Then X follows a discrete uniform distribution.
The probability mass function is given by
$p(x) = \frac{1}{n}$ for $x = x_1, x_2, .., x_n$
$= 0$ otherwise.

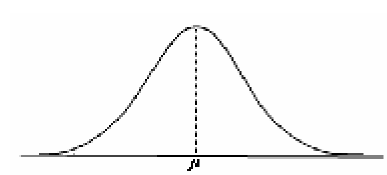
Illustrate with example.
Consider throwing, an unbiassed die once.
The random variable X is the number appearing on the uppermost face.
Now $p(x) = \frac{1}{6}$ for $x = 1, 2, 3, 4, 5, 6$
$= 0$ otherwise.

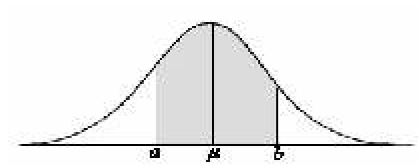
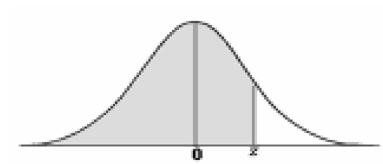
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$p(x)$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td></tr></table>
For a situation to be described using a binomial distribution model,
1. a finite number, n, trials are carried out.
2. the trials are independent.
3. the outcome of each trial is deemed either a success or a failure.
4. the probability, p, of a successful outcome is the same for each trial.
The discrete random variable, X, is the number of successful outcomes in n trial.</td><td></td></tr>
</table>

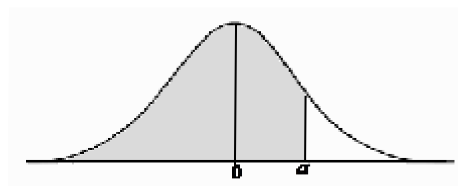
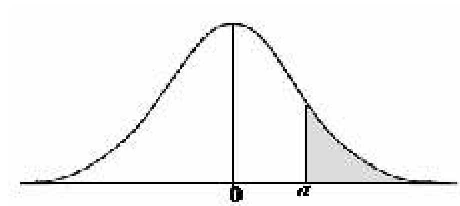
Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		If the above conditions are satisfied, X is said to follow a biomial distribution. This is written X ~ Bin (n, p) The number of trials, n, and the probability of success p, are both needed to describe the distribution completly. p and n are known as the parameters of the binomial distribution. Let X ~ Bin (n, p) Probability mass function is given by $p(x) = P(X = x) = {}^{n}C_{x}(1-p)^{n-x} \cdot p^{x}$, ~ $x = 0, 1, 2, \ldots, n$ $= 0$ otherwise. $E(X) = np$ and $Var(X) = npq$ where $q = 1 - p$ Illustrate with example. Consider throwing an unbiassed die 10 times. Let X be the number of times "6" appears on the uppermost face. Then X Bin $\left(10, \frac{1}{6}\right)$ $p(x) = {}^{10}C_{x}\left(\frac{1}{6}\right)^{x}\left(\frac{5}{6}\right)^{10-x}$,for $x = 0,1,$ 2,3,4,5,6,. . . 10 $= 0$ otherwise.	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	4. Explains Poisson Distribution and calculate probabilities.	1. Events occur singly and at random in a given interval of time or space. 2. λ, the mean of number of occurences in the given interval, is known and finite. The random variable X is the number of occurrences in the given interval. If the above conditions are satisfied, X is said to follow a Poisson distribution, written X ~ $P_0(\lambda)$ $P(X = x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}, x = 0, 1, 2, 3, \ldots$ Note: $P(X = 0) = e^{-\lambda}, P(X = 1) = \lambda \cdot e^{-\lambda}$ $E(X) = \lambda$ and $Var(X) = \lambda$ Illustrate with example. Examples 1. The number of emergency calls received by an ambulance control in one hour. 2. The number of vehicles approaching a particular entry point in a 10 minutes interval.	
	5. Uses poisson distribution as an approximation to the binomial distribution.	When n is large ($n > 50$) and p is small ($p < 0.1$), the binomial distribution X ~ Bin (n, p) can be approximated using a Poisson distribution with the same mean. $X \sim P_0(np)$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
5.11	1. Explains continuous uniform (or rectangular) distribution.	For the continuous uniform (or rectangular) distribution on $[a, b]$ the probability density function is $f(x) = \frac{1}{b-a}$ if $a \le x \le b$ $= 0$ otherwise.  This is written $X \sim U(a, b)$ a and b are said to be the parameters of the distribution. Show that $E(X) = \frac{1}{2}(a+b)$ and $Var(X) = \frac{1}{12}(b-a)^2$ The cumulative distribution function $F(x)$ for a uniform distribution can be found as follows. $X \sim U(a, b)$ $F(t) = P(X \le t) = \int_a^t \frac{1}{b-a}\,dt = \frac{t-a}{b-a}$ Hence $F(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a < x < b \\ 1 & \text{if } x \ge b \end{cases}$	15

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	2. Explains the exponential distribution.	A continuous random variable X having probability density function $f(x)$, given by $f(x) = \lambda \cdot e^{-\lambda x}$ for $x \geq 0$ $= 0$ otherwise, where λ is a positive constant, is said to follow an exponential distribution. λ is parameter of the distribution. [Note: $\int_{-\infty}^{+\infty} f(x)dx = \int_{0}^{\infty} f(x)dx = 1$] $E(X) = \frac{1}{\lambda}$ and $Var(X) = \frac{1}{\lambda^2}$ Show that $P(X>a) = e^{-\lambda a}$ and $P(X > a + b \mid X > a) = e^{-\lambda b}$ $= P(X > b)$	
	3. Explains the normal distribution.	Let X be a continuous random variable. If X is normal distribution with mean μ and standard deviation σ, X has a probability density function, given by $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty$ We write $X \sim N(\mu, \sigma^2)$ The normal distribution curve has the following features.  (i) It is bell shaped. (ii) It is symmetrical about mean (μ) (iii) It extends from $-\infty$ to $+\infty$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		(iv) The maximum value of $f(x)$ is $\frac{1}{\sigma\sqrt{2\pi}}$ (v) The total area under the curve is 1 unit. If X ~ $N(\mu, \sigma^2)$, * approximately 95% of the distribution has within two standard deviation of the mean. * approximately 99.75% of the distribution lie within three standard deviation of the mean. Probability that X lies between a and b is written $P(a < x < b)$ = Area under the normal curve between a and b. 	
	4. Defines the standard normal variable z.	Let X be a normal distribution with mean μ and standard deviation σ X ~ $N(\mu, \sigma^2)$. X is standardised so that the mean is 0 and the standard deviation is 1. Define Z = $\frac{x-\mu}{\sigma}$ Z ~ N (0,1). Z has a probability density function $\phi(Z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$ 	
	5. Uses standard nor-	$P(Z < z) = \phi(z)$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
	mal tables to calculate probabilities.	Using standard normal tables $P(Z< a) = \phi(a)$  $P(Z > a) = 1 - \phi(a)$  Uses the standard normal tables in reverse to find Z, when $\phi(z)$ is given.	
	6. Uses normal approximation to Binomial distribution.	A rule can be used as follow: If $X \sim \text{Bin}(n, p)$ and n and p are such that $np > 5$ and $nq > 5$ (where $q = 1 - p$) then $X \sim N(np, npq)$.	
	7. Explains the continuity correction.	Continuity correction is needed when using a continuous distribution (the normal distribution) as an approximation for a discrete distribution (the binomial distribution). Discuss with examples. Example $P(3<X<5)$ transforms to $P(3.5<X<4.5)$ $P(X < 3)$ transforms to $P(X < 2.5)$ $P(X > 5)$ transforms to $P(X > 5.5)$ $P(X = 4)$ transforms to $P(3.5 < X < 4.5)$	

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
7.	1. Explains what is a Network? 2. Solves problems using Networks.	**Networks** A network can be represented visually by a graph or network diagram consisting of nodes and arcs. Discuss the terminology • Arc • Nodes • Network Use of network techniques • Distribution • Transportation • Financial management • Project planning etc. **Project Management** • What is a project (A project consists of planning, design and implementation of set of tasks leading to accomplishment of a goal such as completed a house or person on the moon. Discuss a small project such as building a house. Identify different activities, preceding activities,i.e. what activities are to be finished before starting another activity etc. • Network representation Discuss how to represent a small project by using a network. Discuss about basic rules. • Explain the following concepts Earliest start time, earliest finish time, Latest start time, Latest finish time and Slack. Discuss how to find the Critical Path. **Maximum Flow Problems** Explain what is a maximal flow problem Many situations can be modeled by a network in which the arcs may be thought of as having a capacity that limits the	20

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		quantity of a product that may be shipped through the arc. In these situations, it is often desired to transport the maximum amount of flow from a starting point (called the source) to a terminal point (called the sink). Such problems are called maximum flow problems. Discuss the solution Algorithm. Minimal spanning tree problem • Explain what is a minimal spanning tree problem. The problem involves choosing the branches for the network that have the shortest total length while providing a route between each pair of nodes. • Explain the nature of the problem by using examples like irrigation system or telecommunication network. • Discuss the solution procedure.	
5.12	Explains Markov Chains.	**Probability** A vector $u = (u_1, u_2, \ldots, u_n)$ is called a **probablity vector** if the all elements of u are non negative and their sum is 1. Example: $u = \left(\frac{1}{3}, 0, \frac{2}{3}\right)$ A square matrix P = (p_{ij}) is called a **stochastic matrix** if each of its rows is a probability vec Example: $P = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} \\ 0 & 1 \end{pmatrix}$	10

Competency Level	Learning Outcomes	Guidelines for subject matter	Number of periods
		Note that if A and B are stochastic matrices of same order then the product AB is a stochastic matrix. A stochastic matrix P is said to be **regular stochastic matrix** if all the elements of some power p^m are positive. Let $P = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, $P^2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$ **Fixed points and regular stochastic matrices (2 x 2)** Let P be a regular stochastic matrix. Then (i) P has a unique fixed probability vector t, and the elements of t are all positive. (ii) the sequence $P, P^2, \ldots$ of powers of P approaches the matrix T whose rows are each the fixed point t. (iii) If p is any probability vector, then the sequence of vectorspP, pP^2, pP^3, ... approaches the fixed point t. Guide students to solve problems on Marcov Chain.	

G.C.E. (Advanced Level) Mathematics

(Implementing from 2009 August)

First Exam under this syllabus will be held in 2011.

The following changes are made in the syllabus.

1. **Allocation of number of periods are changed.**
2. **The section 2.3 (logic) is removed from the syllabus. (Mathematics I)**
3. **Markov Chain (5.12) is introduced in the syllabus (Mathematics II)**

Teachers are kindly requested to follow the changes.

Revised Number of Periods for Mathematics

Mathematics I

Section	Contents	No. of Periods Old	New	Remarks
1.1, 1.2, 1.3	Real numbers	12	12	
2.1, 2.2, 2.4	Set Algebra	20	10	2.3 Removed
2.5, 2.6	Relations	16	18	
3.1, 3.2	Functions in one variable	14	14	
3.3, 3.4	Polynomials	7	7	
3.5, 3.6	Quadratic functions and equations	30	20	
3.7	Rational functions	5	5	
3.8	Exponential functions	6	10	
4.1	Simple algebraic inequalities	10	7	
4.2	Problems involving moduli	10	8	
5.1, 5.2	Permutations and Combinations	27	27	
6	Binomial Expansion	12	12	
7.1,7.2,7.3,7.4	Series	23	23	
8.1, 8.2	Determinants	16	16	
9.1, 9.2, 9.3	Matrices	17	17	
10.1	Trigonomietric Ratio	8	8	
10.2,10.3,10.4	Trigonometric functions, Identities and formulae	17	17	
10.5	Sine and Cosine formulae	8	8	
11.1	Cartesian Coordinates	6	6	
11.2,11.3,11.4, 11.5,11.6,11.7	Straight lines	23	23	
12.1,12.2,12.3, 12.4	Circles	10	12	
13.1,13.2,13.3, 13.4, 13.5	Derivative I	19	26	
13.6, 13.7	Derivative II	10	14	
13.8,13.9,13.10 13.11	Integration	15	21	
13.12,13.13,13.14	Integration	10	14	
	Total	351	355	

Mathematics II

Section	Contents	No. of Periods Old	New	Remarks
1.1, 1.2	Basic of Statistics	10	3	
2.1, 2.2, 2.3,2.4	Presentation of data and information	42	22	
3.1,3.2,3.3 3.4, 3.5	Measures of Central Tendency and Dispersion	46	22	
3.6, 3.7	Skewness	18	03	
4	Index numbers	15	15	
5.1,5.2,5.3, 5.4,5.5	Probability	50	35	
5.6, 5.7,5.8,5.9	Random variables and properties	30	15	
5.10	Probability distributions (discrete)	20	15	
5.11	Probability distributions (continuous)	20	15	
5.12	Markov Chains	0	10	5.12 included
6.1, 6.2	Linear Programnming	18	18	
7	Networks	24	20	
	Total	293	193	

Structure of the Mathematics question papers will be informed by Department of Examination.

Introduction- School Based Assessment

Learning –Teaching and Evaluation are three major components of the process of Education. It is a fact that teachers should know that evaluation is used to assess the progress of learning-teaching process. Moreover, teachers should know that these components influence mutually and develop each other .According to formative assessment (continuous assessment) fundamentals; it should be done while teaching or it is an ongoing process. Formative assessment can be done at the beginning, in the middle, at the end and at any instance of the learning teaching process.

Teachers who expect to assess the progress of learning of the students should use an organized plan. School based assessment (SBA) process is not a mere examination method or a testing method. This programme is known as the method of intervening to develop learning in students and teaching of teachers. Furthermore, this process can be used to maximize the student's capacities by identifying their strengths and weaknesses closely.

When implementing SBA programmes, students are directed to exploratory process through Learning Teaching activities and it is expected that teachers should be with the students facilitating, directing and observing the task they are engaged in.

At this juncture students should be assessed continuously and the teacher should confirm whether the skills of the students get developed up to expected level by assessing continuously. Learning teaching process should not only provide proper experiences to the students but also check whether the students have acquired them properly. For this, to happen proper guiding should be given.

Teachers who are engaged in evaluation (assessment) would be able to supply guidance in two ways. They are commonly known as feed-back and feed- forward. Teacher 's role should be providing Feedback to avoid learning difficulties when the students' weaknesses and inabilities are revealed and provide feed-forward when the abilities and the strengths are identified, to develop such strong skills of the students.

Student should be able to identify what objectives have achieved to which level, leads to Success of the Learning Teaching process. Teachers are expected to judge the competency levels students have reached through evaluation and they should communicate information about student progress to parents and other relevant sectors. The best method that can be used to assess is the S B A that provides the opportunity to assess student continuously.

Teachers who have got the above objective in mind will use effective learning, Teaching, evaluation methods to make the Teaching process and learning process effective. Following are the types of evaluation tools student and, teachers can use. These types were introduced to teachers by the Department of Examination and National Institute of Education with the new reforms. Therefore, we expect that the teachers in the system know about them well

Types of assessment tools:

1.	Assignments	2.	Projects
3.	Survey	4.	Exploration
5.	Observation	6.	Exhibitions
7.	Field trips	8.	Short written
9.	Structured essays	10.	Open book test
11.	Creative activities	12.	Listening Tests
13.	Practical work	14.	Speech
15.	Self creation	16	Group work
17.	Concept maps	18.	Double entry journal
19.	W all papers	20.	Quizzes
21.	Question and answer book	22.	Debates
23.	Panel discussions	24.	Seminars
25.	Impromptus speeches	26.	Role-plays

Teachers are not expected to use above mentioned activities for all the units and for all the subjects. Teachers should be able to pick and choose the suitable type for the relevant units and for the relevant subjects to assess the progress of the students appropriately.The types of assessment tools are mentioned in Teacher's Instructional Manuals.

If the teachers try to avoid administering the relevant assessment tools in their classes there will be lapses in exhibiting the growth of academic abilities, affective factors and psycho-motor skills in the students

Term 1

Group Assignment 1

03.1 Competency Level: Uses various methods for counting.

03.2 Nature : Group Assignment.

03.3 Instructions for the teacher

1. Direct the students to get engaged in this investigation about a week before beginning the lesson on permutations and combination.
2. Instruct students to present the results of the investigation two days before the date scheduled for the lesson.
3. Evaluate the results of the investigation.
4. Begin the lesson on permutation and combination on the scheduled date from the level of their knowledge on permutation.

Note: The terms Principle of counting, permutation, combination and factorial notation should be introduced only after the teacher began the lesson.

03.4 Work sheet

Consider the following phenomenon.

This is an incident that has occured about hundred years ago.

A group of 10 students of a certain school were used to patronise the same canteen daily to have their tea during the school interval. They were in the habit of sitting on the same ten chairs which were in a row. One day the owner of the canteen made the following proposal to them.

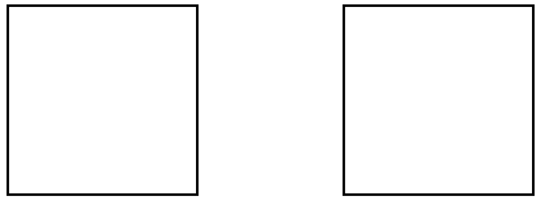
"Today your group is seated in this order. When you come here tomorrow you sit in a different order and likewise change your sitting order daily. You have exhausted all the different orders or sitting I will give you all your refreshments free of charge."

Do the following activity in order to inquire into the canteen owner's proposal mathematically.

(i) Take 5 pieces of equal square card boards and mark them as A, B, C, D and E as shown below:

A	B	C	D	E

(ii) Draw two squares a little bigger than the above squares on a sheet of paper in a row.



In how many different ways can the two squares marked A and B can be placed inside the two squares on the sheet of paper.

(iii) a) Drawing three square in a row and using the cards A, B and C.

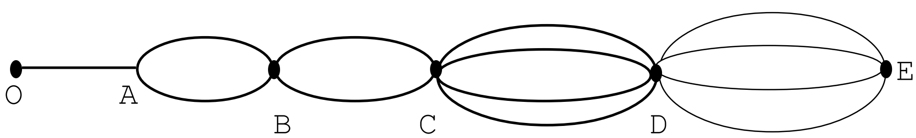
b) Drawing four squares in a row and using the cards A, B, C and D.

c) Drawing five squares in a row and using the cards A, B, C, D and E.

Find the number of different ways in which the cards can be placed with one card inside a square.

Note down the results of each of the cases above on a sheet of paper.

2. The network of a system of roads connecting the 5 cities A, B, C, D and E to a city O is as follows:



(a) In how many different ways can (i) A (ii) B (iii) C (iv) D (v) E can be reached from O?

(b) Describe a convenient way of obtaining the above results.

(c) Is there a relationship between these results and the results obtained in the activity (1) above.

If there is a relationship explain why it is so.

3. (a) Write an expression as a product of integers which gives the number of different ways in which 10 different objects (living, non-living or symbolic) can be placed in a row.

Simplify this expression. Hence write down your judgement with regard to the proposal made by the canteen owner mentioned earlier.

Write an expression in the form of a product for the number of different ways in which n different objects can be arranged in a row.

Criteria for Evaluation

1. Engaging in the task as instructed.
2. Revealing mathematical relationships.
3. Construction of mathematical models.
4. Reaching conclusions.
5. Expressing ideas logically.

Group Assignment 2

Nature of the student based activity: Open text assignment.

04.1 Competency Level:

04.1.1 Interprets the events of a random experiment.

04.1.2 Applies probability models for solving problems on random events.

04.2 Nature of the assignment : Open text assignment of revising the knowledge about sets and probability.

04.3 Instructions for the teacher

1. About 2 weeks before beginning the lesson on probability instruct the students to study the lessons on sets and probability in the text books from grades 6 to 11. Give the given assignment to the students.
2. Instruct them to submit answers about one week before the beginning of the lesson.
3. After evaluation of the answers begin the lesson providing the necessary feedback.

Assignment

(1) (i) Write all subsets of A = {1, 2, 3, 4, 5 }. How many subsets are there ?

(ii) Select the subsets of B = $\{x \mid x \in \mathbb{Z}^+,\ x < 10\}$ from the following sets.

P = { 1, 4, 9, 16} Q = { 2, 3, 5, 7 }

R = { Prime numbers less than 10 } S = {Counting numbers less than 10}

T = { 2, 4, 6, 8 } U = {1, 2, 3, 4, 5, 6, 7, 8, 9 }

Out of the subsets you have selected write down the proper subsets of A, if any.

(2) If $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 3, 5, 7, 9\}$ and $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, write the elements of

(i) $A \cap B$ (ii) $A \cup B$ (iii) A' (iv) B' (v) $A' \cap B'$

(vi) $A' \cup B'$ (vii) $(A \cap B)'$ (viii) $A \cap B'$ (ix) $(A \cup B)'$ (x) $A' \cap B$

(3) State the following laws about set algebra and varify them by means of Venn diagrams.

(i) Commulative Law (ii) Distributive Law

(iii) Associative Law (iv) De Morgans Laws

(4) Underline the correct results out of the folowing.

(i) $A \cap \phi = A$ (ii) $A \cup \phi = A$ (iii) $\mathcal{E} \cap \phi = A$

(iv) $A' \cup A = A$ (v) $A' \cap A = \phi$

(5) (i) Define a random experiment.

(ii) Select random experiments from the following:

(a) Sun will rise tomorrow.

(b) Testing the top side when a coin is tossed.

(c) Testing the top side when a dice marked from 1 to 6 is tossed.

(d) Testing the number of sick students sent home during school hours.

(e) Measuring the life span of an electric bulb.

(f) Drawing a ball at random from a bag containing 3 red balls and 1 blue ball which are identically equal.

(iii) Write the sample space of the random experiments you have selected above.

(6) In the random experiment of observing the top sides when two coins are tossed simultaneously.

(i) Write the sample space.

(ii) Write two simple events in it.

(iii) Write two composite events in it.

(7) What are mutually exclusive events. Explain with an example.

(8) Toss a coin 25 times and complete the following table.

Number of Times	Side obtained (Head or Tail)
1	
2	
3	
.	
.	
.	
25	

(i) Find the success fraction of obtaining a head when the coin is tossed 25, times.

(ii) Repeat the experiment 50 times, 100 times and find the success fraction of obtaining a head.

(iii) If success fraction is to be taken as a measure of probability how should be the number of times the experiment is to be repeated?

(9) What is an equally probable event? Select equally probable events from the following random experiments.

(i) Observing the side obtained when a coin is tossed.

(ii) Observing the side obtained when an unbiased dice marked 1-6 is tossed.

(iii) Observing the colour of a ball taken randomly from a bag containing 2 blue balls and 3 red balls.

(iv) Observing the number of a card taken randomly from a set of identical cards numbered from 1-9.

(10) (i) Write the sample space for the random experiment (ii) above.

If A = { Obtaining an even number}

B = { Obtaining a prime number }

C = { Obtaining a square number }

D = { Obtaining an odd number }

(ii) Find

(a) P (A) (b) P (B) (c) P(C) (d) P(D) (e) $P(A \cap B)$

(f) $P(A \cap C)$ (g) $P(C \cap A)$ (h) $P(A \cup B)$ (i) $P(A \cup B \cup C)$ (j) $P(A \cap B \cap C)$

(iii) Prove that

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(b) $P(A \cup B \cup C) = P(A) + P(B) - P(C) - P(A \cap B) - P(B \cap C)$

$- P(C \cap A) + P(A \cap B \cap C)$

(iv) (a) Select two mutually exclusive events.

(b) Find $P(A \cup D)$

Criteria for Evaluation

1. Use of text books for obtaining the necessary knowledge.
2. Knowledge of set Algebra.
3. Knowledge of basic concepts in probability.
4. Following the given instructions correctly.
5. Expressing ideas freely.

For the written test teacher can choose questions from the following or he / she can prepare questions on his/ her own.

Permutations & Combinations

1. (a) Three boys and three girls sit in a row of six seats.

   Find the number of ways that

   (i) they can occupy the seats.

   (ii) the three girls sit together,

   (iii) the three girls and the three boys sit in alternate seats.

   (b) In a certain examination you are required to answer six out of nine questions.

   Find the number os ways that you can choose the six questions.

   Also, find the number of ways that you can choose the six questions,

   (i) if the first three questions are compulsory.

   (ii) if at least four should be selected from the first five questions.

2. (a) a committee consists of 3 Mathematics teachers and 4 Biology teachers. In how many ways can they sit in a row if

   (i) they may sit in any order

   (ii) the teachers of the same subject sit next to each other.

   (iii) no two teachers of the same subject sit next to each other.

   (iv) the teachers of the same subject sit next to each other such that one particular Mathematics teacher always sit with his wife who is a Biology teacher.

(b) Consider a regular polygon with n sides.

   (i) How many diagonals are three in the polygon?

   What is the value of n, if the number of sides is twice the number of diagonals?

   (ii) How many triangles are there, whose vertices are the vertices of the polygon?

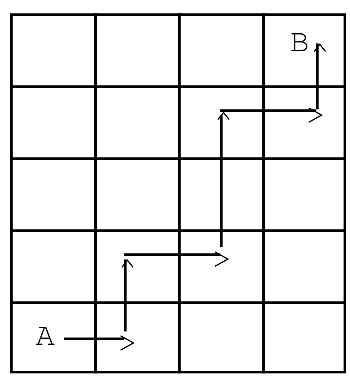
   (iii) Out of the triangles in (ii) above, how many have exactly one side coincident with a side of the polygon?

   (iv) Out of the triangles in (ii) above, how many have two sides coincidentr with two sides of the polygon?

   Deduce that, if $n > 3$, the number of triangles whose vertices ar e the vertices of the polygon and the sides are the diagonals of the polygon is

$$\frac{n}{6}(n-4)(n-5)$$

3. (a) A rectangular corridor is paved with 20 tiles as shown in the diagram. A little girl wishes to go from tile A to tile B, by jumping from one tile to a neighbouring tile on the right or a neighbouring tile in front (one such possibility is shown in the diagram) In how many ways can she do this?



(b) A group of children consists of 3 girls and 2 boys. A second group of children consists of 2 girls and 3 boys and a third group of children consists of 1 girl and 4 boys. A team of three children with at most two from a group is selected at random. In how many ways can the team be selected so that always there is one girl and 2 boys in the team.

Derivatives II

1 (a) A water tank has the shape of a frustum of a right circular cone. The height of the tank is 5 metres, and the radii at the top and bottom are 2 metres and 1 metre respectively. Water is being pumped at a constant rate 0.7 cubic meters per minute into the tank, which was initially empty. Show that, when the height of water level from bottom is x metres ($0<x<5$), the volume of water in the tank is $\frac{\pi}{75}\left(x^3+15x^2+75x\right)$ cubic metres. Find the rate at which the height of water level is increasing when $x = 2$.

(b) Let $f(x) = x^3 - 2x^2 + cx + d$, where c and d are constants. The graph of $y = f(x)$ passes through the point (1, 4) and the tangent at this point to the curve is parallel to x axis. Find the value of c and d.

Also, find

(i) the range of values of x for which y is increasing,

(ii) the range of values of x for which y is decreasing,

(iii) the co-ordinates of maximum and minimum points of the graph.

Sketch the graph of $y = f(x)$.

2 (a) A window has the shape of a rectangle surmounted by a semicircle. The total perimeter of the window is 20 m. Find the dimensions of the window such that the total area of the window is maximum.

(b) Find the maximum and minimum points of $y = \frac{3x^2-3}{6x-10}$. Sketch the graph of $y = \frac{3x^2-3}{6x-10}$.

Draw the graph of $xy = 1$ in the same diagram. Hence, show that $3x^2 - 9x + 10 = 0$ has only one real root and this root is less than -1.

Mathematics

Integration

1 (a) Find the constants A, B and C such that $\dfrac{1}{x(2x-1)^2} = \dfrac{A}{x} + \dfrac{B}{2x-1} + \dfrac{C}{(2x-1)^2}$

Hence find $\int \dfrac{1}{x(2x-1)^2}\,dx$

(b) Use a suitable substitution to evaluate $\int_0^2 x(2-x)^8\,dx$ [Hint : put $n = 2-x$]

(c) Write $\sin 3x \sin x$ in the form k(cosC–cosD) where k is a constant. Hence, find $\int \sin 3x.\sin x\,dx$

(d) Evaluate $\int_0^{\frac{\pi}{2}} \dfrac{\sin x}{3+5\cos x}\,dx$ [Hint : put $3+5\cos x = u$]

$$\int \frac{1}{x^4-1}\,dx$$

2. (a) Find the constants A, B and C such that $\dfrac{1+x^2}{x(1-x)} = A + \dfrac{B}{x} + \dfrac{C}{1-x}$

Hence, show that $\int_2^3 \dfrac{1+x^2}{x(1-x)}\,dx = \ln\dfrac{3}{8} - 1$

(b) Use a suitable transformation to find $\int \cos^{10} x.\sin^3 x\,dx$ [Hint : put $u = \cos x$].

(c) Show that $\cos 4x = 8\cos^4 x - 8\cos^2 x + 1$

Hence or otherwise, find $\int \cos^4 x.dx$

3. (a) Let $f(x) = \frac{1}{x^4 - 1}$

Find the constants A, B, C and D such that $\frac{1}{x^4 - 1} = \frac{A}{x - 1} + \frac{B}{x + 1} + \frac{Cx + D}{x^2 + 1}$

Hence, find $\int \frac{1}{x^4 - 1} dx$

(b) Using the identity $\cos x = 2\cos^2 \frac{x}{2} - 1,$

Evaluate $I = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \cos x} dx$

Use the substitution $x = \frac{\pi}{2} - y,$ and show that

$J = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \sin x} dx = I$ and write the value of J.

(c) Use a suitable substitution and evaluate

$$\int_0^{\frac{\pi}{2}} \frac{1 - \sin x}{(x + \cos x)^2} dx$$

[Hint : put, $x + \cos x = u$)

Term 2
School based assessment
Instrument number 01.

1.1 Competency Level : Solves integration problems using the method of integration by parts.

1.2 Nature of Instrument : An individual assignment for derivation and use of the formula for integration by parts.

1.3 Instructions for the teacher for the implementation of the instrument

1. Provide the given work sheet for the students and get them engaged in the task.
2. Direct the tudents to obtain the final answer to the problem by the successive application of the formula or by any other technique.
3. Provide the necessary feedback after evaluation of the assignment.

1.4 Qualitative applications (necessary instruments): Copies of work sheet.

1.5 Work sheet

You are required to get engaged in the task by following the instructions given below.

1. When u and v are differentiable functions of x we know that

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

According to the definition of the antiderivative of a function evaluate $\int\left(u\frac{dv}{dx} + v\frac{du}{dx}\right)dx$

Using the laws on integration, derive that $\int u\left(\frac{dv}{dx}\right)dx = uv - \int v\left(\frac{du}{dx}\right)dx + c$

2. Using the result you obtained above evaluate the following integrals.

(i) $\int x\sin x\,dx$; Take $u = x$, $\frac{dv}{dx} = \sin x$

(ii) $\int x^2\cos x\,dx$; Take $u = x^2$, $\frac{dv}{dx} = \cos x$

(iii) $\int e^x\sin x\,dx$; Take $u = e^x$, $\frac{dv}{dx} = \sin x$ or

$$u = \sin x, \quad \frac{dv}{dx} = e^x$$

(iv) $\int \ln x \, dx;$ Take $u = \ln x, \quad \frac{dv}{dx} = 1$

1.6 Criteria for Estimation

1. Derivation of the formula for integration by parts.
2. Use of this formula.
3. Derivation of integrals of the relevant functions as anti-derivatives.
4. Obtaining the final results.
5. Following the given instructions.

1.7 Marks for criteria

1. Very Good - 4 marks
2. Good - 3 marks
3. Fair - 2 marks
4. Should improve- 1 mark

1.8 Maximum marks that can be earned for this instrument : 4 x 5 = 20 marks

Term 2

School based assignment

Instrument number 02.

2.1 Competency Levels : 5.7 Analyses the properties of probability distributions of a continuous variable and a discrete variable.

5.8 Interprets the mathematical expectation of a random variable.

2.2 Nature of Instrument : An individual assignment for finding the probability distribution, mean, variance and moment of a random variable.

2.3 Instructions for the teacher for the implementation of the instrument

1. After the lesson on probability distributions, get the students engaged in this assignment to test whether those concepts are instilled in them.
2. Give the necessary feed back after evaluating the assignment.
3. Provide the necessary feedback after evaluation of the assignment.

2.4 Qualitative applications (necessary instruments): Copies of work sheet.

2.5 Work sheet

You are required to get engaged in the task by following the instructions given below.

1. (i) Define the probability distribution of a discrete random variable X and state its special properties.

   (ii) Define , the mean of X [Expected value or E (X)]

   (iii) The probability distribution of a discrete random variable X is given below.

X	-1	0	1
P(x)	k^2	$-\frac{k}{2}$	$\frac{1}{2}$

   Find the value which k can take.

   (iv) Find E(X).

   (v) Write down the probability distribution of 2X + 1.

   (vi) Find E (2X + 1) using the distribution in (v) above.

   (vii) Varify that E (2X + 1) = 2E (X) + 1

   (viii) Write down the probability distribution of X^2.

   (ix) Find $E(X^2)$ using the distribution in (viii) above.

   (x) Define Var (X).

   (xi) Find Var (X) using that definition.

   (xii) Varify that Var (X) = $E(X^2) - [E(X)]^2$

   (xiii) Explain what is meant by the first moment of a random variable about the origin.

   (xiv) What is the second moment of a random variable about the mean?

2. (i) Define the probability density function of a continuous random variable X. State its special properties.

(ii) Define the mean [expected value of X or E(X)] of X.

(iii) The probability density function of a continuous random variable X is given below.

$$f_X(x) = kx \; ; \quad \text{for} \quad 1 \le x \le 3$$
$$\qquad 0 \; ; \qquad \text{else.}$$

Find the possible values of k.

(iv) Find E (X).

(v) Write down the probability density function of 2X + 1.

(vi) Find E (2X + 1) using the function in (v) above.

(vii) Varity that E (2X + 1) = 2E (X) + 1.

(viii) Write down the probability density function of X^2.

(ix) Find E (X) using the function in (viii) above.

(x) Define Var (X).

(xi) Find Var (X) using the definition in (X) above.

(xii) Varify that Var (X) = E (X^2) - [E (X)]^2

(xiii) Explain what is meant by the first moment of a random variable about the origin.

(xiv) Whaty is the second moment of a random variable about the mean.

2.6 Criteria for Estimation.

1. Expressing definitions.
2. Use of properties of a probability distribution.
3. Use of definitions for the expectation and variance of a random variable.
4. Finding the expectation of a function defined on a random variable.
5. Varification of a given result.

2.7 Marks for Criteria

Very Good	-	04 marks
Good	-	03 marks
Fair	-	02 marks
Should improve	-	01 mark

2.8 Maximum marks for the instrument 5 x 4 = 20 marks.

For the written test teacher can choose questions from the following or he / she can prepare questions on his / her own.

Binomial Expansion

1 (a) Find the coefficient of x^{32} and x^{-17} in the expansion of $\left(x^4 - \frac{1}{x^2}\right)^{15}$

(b) In the binomial expasion of $\left(1 + \frac{x}{n}\right)^n$ in ascending powers of x, the coefficient of x^2 is $\frac{7}{16}$. Given that n is a positive integer,

(i) find the value of n.

(ii) evaluate the coefficient of x^3 in the expansion.

2. (a) Expand $(1 + ax)^8$ in ascending powers of x upto and including the term in x^2

The coefficients of x and x^2 in the expasion of $(1+ bx)(1+ ax)^8$ are 0 and -36 respectively. Find the values of a and b given that $a > 0$ and $b < 0$.

(b) Find the term independent of x in $\left(1 + x + 2x^3\right)\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$

3. (a) If x is so small that x^3 and higher powers of x are negligible, show that $(3 + 2x)\left(3 - \frac{x}{3}\right) \cong 3^8\left(9 - 3x - 2x^2\right)$

(b) Given that the coefficient of x in the expansion of $\left(1 + ax\right)^5$ is equal to the coefficient of x^4 in the expansion of $\left(9 + \frac{x}{3}\right)^6$ calculate the value of a.

Integration

1 (a) Use integration by parts to evaluate $\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$.

(b) Find the area enclosed by the curves $y = -(12 - 8x + x^2)$ and $y = x$.

(c) The table below gives the value of a function.

x	1	1.5	2	2.5	3
$f(x)$	0.8	1.2	1.7	2.3	3.0

Evaluate $\int_1^3 f(x) dx$ (i) Using trapezoidal rule with 4 intervals.

(ii) Using simpson's rule with 4 intervals.

2. (a) Using integration by parts show that $\int_1^2 x^2 \ln x \, dx = \frac{8}{3}\ln 2 - \frac{7}{9}$

(b) Varify that $y^2 = 3x$ and $x^2 = 3y$ passes through the point (3,3) and find the area of the finite region bounded by these curves.

(c) Evaluate $\int_1^5 \frac{1}{x^2} dx$

Find approximate value of $\int_1^5 \frac{1}{x^2} dx$ using simpson's rule with 4 intervals.

3. (a) Use integration by parts to find $\int x \cdot e^{3x} dx$

(b) Calculate the area of the finite region bounded by the curve $y = x\,(3-4x)$ and the line $y = x$.

Inequalities involving moduli

1 $|x| < a$ if and only if $-a < x < a$

$|x| > a$ if and only $x < -a$ or $x > a$ where $a > 0$.

Using the above results or otherwise, find the set of values of x satisfying the following inequalities.

(a) $|3-2x| < 5$ (b) $|2x+3| > 1$ (c) $|x-4| > 2x-2$

2 Find the set of values of x for which

(a) $|x-2| - 2|2x-1| > 0$

(b) $x > |3x-8|$

3 (a) Find the set of values of x satisfying the inequality.

$$x + 2|x-1| > 2|x+1| - 3$$

(b) Draw the graphs of (i) $y = x^2 - x - 6$ and (ii) $y = |x^2 - x - 6|$ in the same diagram.

4 Draw the graph of $y = |x^2 - 4x + 3|$ and $y = |x-1|$ in the same diagram.

Hence solve the inequality $|x^2 - 4x + 3| > |x-1|$

Series

1 (a) Find $\sum_{r=1}^{n} \log 2^r$

(b) Let $S_n = 1 + 2x + 3x^2 + \ldots + nx^{n-1}$

By considering $(1-x)S_n$ find S_n

Hence, when $|x| < 1$, deduce $\sum_{n=1}^{\infty} nx^{n-1}$

(c) Let $f(r) = \dfrac{1}{r^2}$ and $U_r = \dfrac{2r+1}{r^2(r+1)^2}$

Show that $U_r = f(r) - f(r+1)$

Hence find $\sum_{r=1}^{n} U_r$ and show that $\sum_{r=1}^{\infty} U_r$ is convergent . Let $S_n = \sum_{r=1}^{n} U_r$. Find the minimum value of n for which .

Statistics

1 (a) Two uniform dice are thrown. Obtain the probability distribution for higher of the two scores (or common score if both are equal).

$X = x$	1	2	3	4	5	6
$P(X = x)$						

$S_n \quad \dfrac{9999}{10000}$

Varify that $\sum_{x=1}^{6} P(X = x) = 1$

Find

(i) the mode

(ii) the mean

(iii) the variance

(iv) $P(X < 3)$

(v) $P(X \quad 3)$

(b) A random variable X has probability density function

$$f(x) = kx^2(2 - x) \quad \text{if } 0 \le x < 2$$
$$= 0 \qquad \text{otherwise.}$$

Find

(i) the value of k. (ii) E (X)

(iii) Var (X) (iv) the mode (v) $P(1 < X < 2)$

References

- Bostock, L. and Chandler, L. Pure Mathematics I
  Stanley Thrones (Publishers) Ltd., 1993

- Bostock, L. and Chandler, L. Pure Mathematics II
  Stanley Thrones (Publishers) Ltd., 1993

- Crawshaw, J. and Chambos, J., A concise couse in A-Level Statistics
  ELBS, Stanley Thrones (Publishers) Ltd., 1992

- උසස් පෙළ සංකරණ සංයෝජන, සම්භාවිතාව සහ සංඛ්‍යානය, අධ්‍යාපන ප්‍රකාශන දෙපාර්තමේන්තුව - 1996

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