G. C. E. (Advanced Level)

PHYSICS

Grade 12

Resource Book

Unit 3: Oscillations and Waves

Department of Science

Faculty of Science and Technology

National Institute of Education

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Physics

Resource Book

Grade 12

Unit 3: Oscillations and Waves

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Message from the Director General

The National Institute of Education takes opportune steps from time to time for the development of quality in education. Preparation of supplementary resource books for respective subjects is one such initiative.

Supplementary resource books have been composed by a team of curriculum developers of the National Institute of Education, subject experts from the national universities and experienced teachers from the school system. Because these resource books have been written so that they are in line with the G. C. E. (A/L) new syllabus implemented in 2017, students can broaden their understanding of the subject matter by referring these books while teachers can refer them in order to plan more effective learning teaching activities.

I wish to express my sincere gratitude to the staff members of the National Institute of Education and external subject experts who made their academic contribution to make this material available to you.

Dr. (Mrs.) T. A. R. J. Gunasekara
Director General
National Institute of Education
Maharagama.

Message from the Director

Since 2017, a rationalized curriculum, which is an updated version of the previous curriculum is in effect for the G.C.E (A/L) in the general education system of Sri Lanka. In this new curriculum cycle, revisions were made in the subject content, mode of delivery and curricular materials of the G.C.E. (A/L) Physics, Chemistry and Biology. Several alterations in the learning teaching sequence were also made. A new Teachers' Guide was introduced in place of the previous Teacher's Instruction Manual. In concurrence to that, certain changes in the learning teaching methodology, evaluation and assessment are expected. The newly introduced Teachers' Guide provides learning outcomes, a guideline for teachers to mould the learning events, assessment and evaluation.

When implementing the previous curricula, the use of internationally recognized standard textbooks published in English was imperative for the Advanced Level science subjects. Due to the contradictions of facts related to the subject matter between different textbooks and inclusion of the content beyond the limits of the local curriculum, the usage of those books was not convenient for both teachers and students. This book comes to you as an attempt to overcome that issue.

As this book is available in Sinhala, Tamil, and English, the book offers students an opportunity to refer the relevant subject content in their mother tongue as well as in English within the limits of the local curriculum. It also provides both students and teachers a source of reliable information expected by the curriculum instead of various information gathered from the other sources.

This book authored by subject experts from the universities and experienced subject teachers is presented to you followed by the approval of the Academic Affairs Board and the Council of the National Institute of Education. Thus, it can be recommended as a material of high standard.

Dr. A. D. A. De Silva
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Content

Chapter - 01

Oscillations

Introduction

Have you experienced swaying in a swing. You were swaying to-and-fro. This we call as an oscillation as you were swaying to-and-fro around a middle point. In this chapter we hope to analyze this motion. Also you may have observed the two way vibrations of the prong of a vibrating tuning fork or a hacksaw blade clamped at one end. We call this as vibrations since whole object is swaying to-and-fro. In this case each part of the object is swaying with the same frequency but with different amplitudes.

Periodic motion

A motion which repeats itself at equal intervals of time is a periodic motion.

Eg.:- Motion of all planets around the sun.

Oscillatory motion

To-and-fro motion of a body around a mean position is an oscillatory motion.

Eg.:- Oscillation of a cradle or a swing.

Simple harmonic motion

If the acceleration of a moving body is always directed towards an equilibrium point, its magnitude being directly proportional to the distance from the same point, then that motion of the body is called simple harmonic motion. Simple harmonic motion (SHM) is essentially an oscillatory motion, but not all oscillations are simple harmonic.

Characteristics of simple harmonic motion

i. The motion is periodic.

ii. The acceleration of the moving body is proportional to the displacement from the mean position (or equilibrium position).

iii. The acceleration is directed towards the mean position.

Before a theoretical study of simple harmonic motion, it is necessary to enquire it qualitatively. For this purpose let us consider small oscillations of a pendulum consisting of a bob hung by a light weight inextensible string. The upper end of the string is attached to a fixed point. When the bob is pulled aside slightly and released it would oscillate to-and-fro in a vertical plane along a circular path.

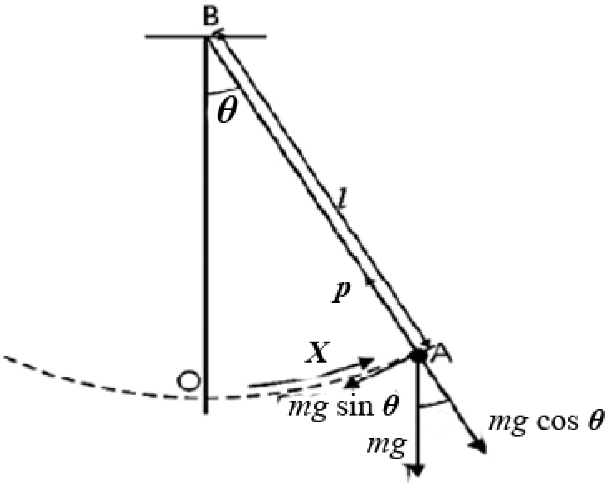


Figure 1.1-Simple pendulum

Considering this motion, when the bob is in a certain position A, $OA = x$ and $O\hat{B}A = \theta$. The forces acting on it are the tension *T* of the string and weight *mg*. At point A, when the weight acting vertically downwards is resolved radially and tangentially. The tangential component $mg\sin\theta$ can be shown as the unbalanced restoring force acting towards O.

The equation of motion of the bob can be expressed as,

$$-mg\sin\theta = ma$$

where '*a*' is the acceleration of the bob along the tangent drawn at A. The force is towards "O" and the displacement *x* from O along the arc is measured in the opposite direction which is therefore given the negative sign. When θ is small (not exceeding 10^0) $\sin\theta \cong \theta(rad)$

$$-mg\theta = ma$$

$$-mg\frac{x}{l} = ma$$

$$a = -\frac{g}{l}x$$

$$a = -\omega^2 x$$

$$\omega^2 = \frac{g}{l}$$

This relation represents simple harmonic motion for small amplitudes.

If *T* is the periodic time of the simple pendulum

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

T depends only on the length of the pendulum. At the location of the pendulum, *g* is considered to be a constant. The amplitude of the oscillation gradually decreases due to air resistance. This expression was derived first by Galileo. He had used his pulse to measure the time of oscillation.

Equations related to simple harmonic motion

Consider a body performing circular motion along a circle of radius *r* and centre Z with a uniform angular velocity ω. CZF is a fixed diameter. The foot of the perpendicular projection (M) from the moving body on to this diameter moves from Z to C, and passing through Z again and moving to F returns to Z. During this period of time the body performs one complete circular motion (cycle) along the circumference starting from O in anticlockwise direction. The to-and-fro motion of the foot of the projection along CZF can be shown to perform simple harmonic motion.

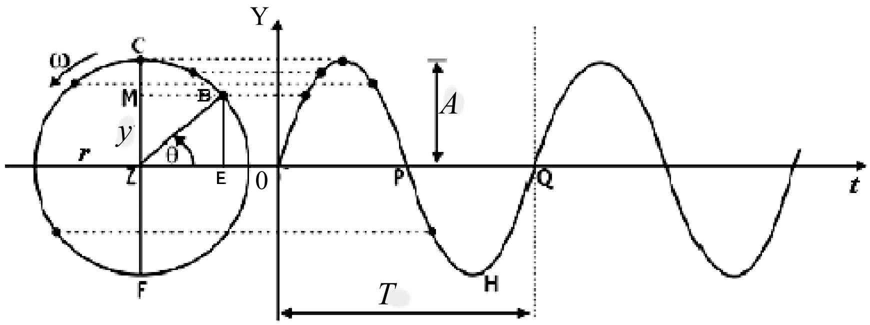


Figure 1.2 - Simple harmonic curve

Let M be the foot of the perpendicular projection from B to CZ at a certain instant when $EZB = \theta$, during the circular motion of B. When the body is at B, the acceleration $\omega^2 r$ of the body is directed towards the radius BZ. Then the acceleration of M towards Z will be $\omega^2 r \sin\theta$

Taking $r\sin\theta = MZ = y$

Acceleration of M towards is $\omega^2 y$

Since ω^2 is a constant,

Acceleration of M towards Z $\propto$ distance from Z to M

Hence acceleration towards Z is $-\omega^2 y$

This equation shows that the acceleration of M towards Z is directly proportional to the distance from Z to M. The graph of acceleration against displacement is a straight line with a negative gradient.

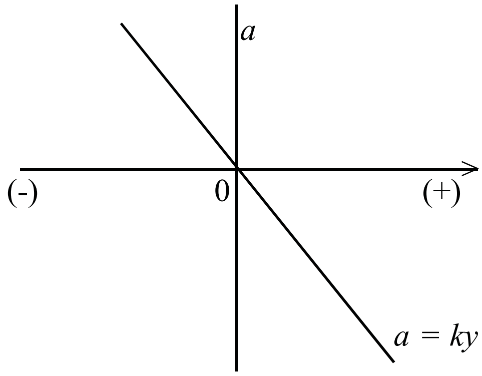


Figure 1.3 - Graph of acceleration against displacement in SHM

Period

The time taken by the foot of the perpendicular to move from C to F and then return to C is called the period (T). During this period of time the body completes one round (one cycle) along the circumference. The angle covered is 2π and if ω is the angular velocity, it can be written as,

$$T = \frac{2\pi}{\omega}$$

ZC or ZF which is the maximum distance from Z to the foot of the projection is known as the amplitude of motion. According to the diagram it is equal to the radius, ZC = r = A of the circle. Next let us consider the variation of the distance from Z to the foot of the projection with time t.

$$y = ZB \sin\theta$$
$$ZB = A \text{ and } \theta = wt \Rightarrow y = A \sin wt$$

This is the equation of the above curve and it has a sinusodal shape. 'A' is the maximum displacement and callted amplitude. When the object moves f number of times per second around the circle. The point M oscillates f times per second alone CF, therefore $T = \frac{1}{f}$ and $\omega = 2\pi f$

The velocity of a body performing simple harmonic motion

When a body performs circular motion, its velocity ω at a certain instant of time acts along the tangent drawn at point B to the circular path as shown in Figure 1.4

At this instant the velocity of M along FC is,

$v = r\omega\cos\theta$ according to trigonometry, $(\sin\theta)^2 + (\cos\theta)^2 = 1$

But $y = r\sin\theta$

$$\left(\frac{y}{A}\right)^2 + \left(\frac{v}{A\omega}\right)^2 = 1$$
$$v^2 = \omega^2\left(A^2 - y^2\right)$$
$$v = \pm\omega\sqrt{A^2 - y^2}$$

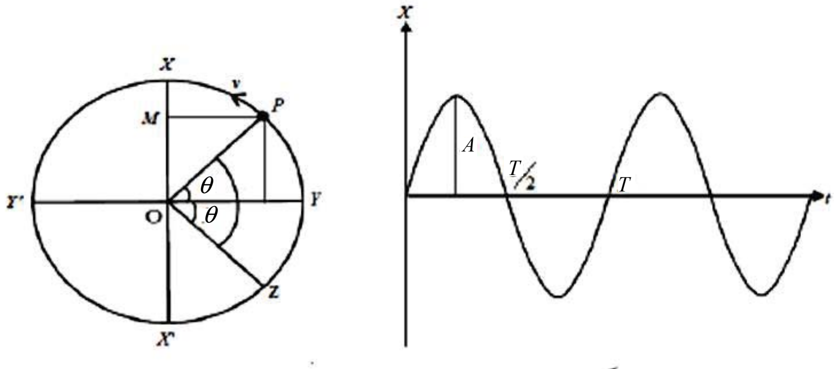


Figure 1.4 - Circular representation of SHM starting away from the centre

As given in the Figure 1.4 if the body is started at point Z instead of point Y then the measurement of time starts from Z (t = 0). During time t the angular displacement is ωt.

Then $Y\hat{O}P = Z\hat{O}P - Z\hat{O}Y = \omega t - \phi$ After time t the coordinates of M are,

$x = A\sin(\omega t - \phi)$ and $y = A\cos(\omega t - \phi)$

$(\omega t - \phi)$ is called **phase** of the vibration and $-\phi$ is the **initial phase**.

If the osillation started at a phase in front then, $x = A\sin(\omega t + \phi)$ and $y = A\cos(\omega t + \phi)$.

The quantity $(\omega t - \beta)$ which varies with time is known as the **phase** while β is called the **phase** angle.

If the oscillation started at a phase in front, then

$$y = A\sin(\omega t + \beta)$$

Phase

The phase can be referred to as the state of motion at any instant of a particle performing simple harmonic motion.

Oscillating Systems - Spring and mass

Suppose a body of mass *m* can move without friction along a straight line on a flat smooth surface. The body is attached to one end of a spring, the other end of which is held stationary. The position of the body is described in relation to the coordinate *x*, taking the origin ($x = 0$) of this coordinate system as the equilibrium position, at which location of the spring is neither stretched nor compressed. When the body is displaced to the right, *x* becomes positive, the spring is stretched and it exerts a force on the body towards the left (along the negative *x* direction) or towards the equilibrium position. When the body is displaced to the left, *x* becomes negative, the spring is compressed and it exerts a force on the body towards the right along the positive *x* direction, again towards the equilibrium position. Thus the sign of the *x* component of the force on the body is always opposite to that of *x* itself. If in addition, the spring obeys Hooke's law, then the force, *F*, on the body is given by,

$$F = -kx$$

where k is a constant for the spring and is in general, called the spring constant.

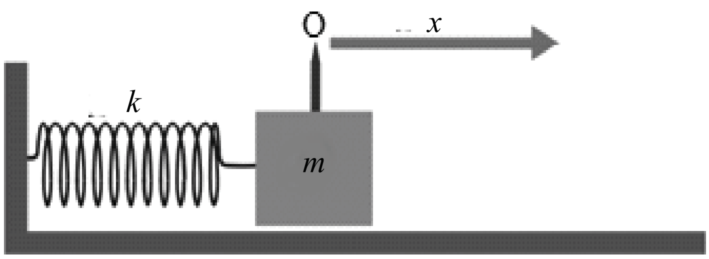


Figure 1.5 - SHM of a mass under the spring force obeying Hooke's law

Now suppose the body is displaced a small distance *x*, and released. The spring exerts a restoring force on the body, the body accelerates in the direction of this force and moves towards the equilibrium position with increasing speed. The rate of this increase (i.e., the acceleration) however, is not constant, since the accelerating force becomes smaller as the body approaches the equilibrium position. When the body reaches the centre, the restoring force decreases to zero but because of the velocity (or the kinetic energy) the body has acquired, it over shoots the equilibrium position and continue to move towards the left.

As soon as the body passes the equilibrium position, the restoring force comes into play, directed now towards the right. The speed of the body thus decreases, at a rate that increases with increasing distance from O. It therefore, comes to rest at some point to the left of O, and repeats its motion in the opposite direction. This motion is confined to a range $\pm A$ on either side of the equilibrium position, each to and fro movement taking place in the same interval of time.

The motion discussed above is under the influence of an elastic restoring force, which is proportional to the displaacement and in absence of all friction with no loss of energy, would continue indefinitely, once it has been started. Therefore, it is a simple harmonic motion.

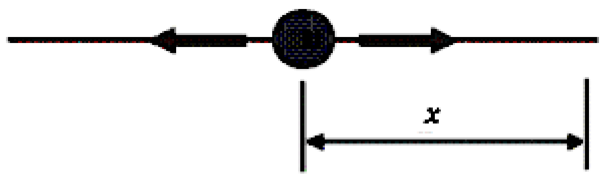


Figure 1.6 - SHM of mass m

A complete cycle of motion means one round trip say moving the mass *m* from O to Q then Q to P and then P to O. See the diagram in Figure 1.7. The amplitude *A*, is the maximum displacement from the equlibrium position or in other words, the maximum value of $|x|$.

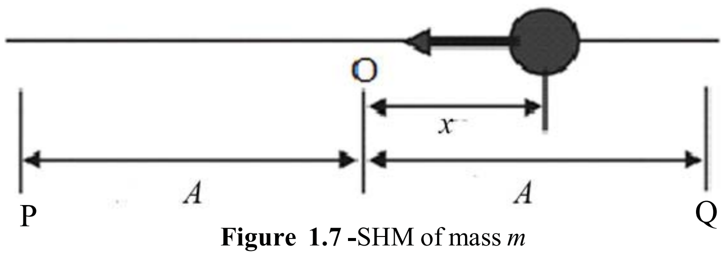


Figure 1.7 -SHM of mass *m*

The Figure 1.7 represents a vibrating body at some instance when the displacement of a particle from its equilibrium position O is described by the coordinate *x*. The mass of the body is *m* and the resulting force acting on it is the elastic restoring force $-kx$.

From Newton's second law ,

$$F = -kx$$

$$= ma$$

$$\therefore a = -\frac{k}{m}x$$

$$a = -\omega^2 x$$

$$\omega^2 = \frac{k}{m}$$ where, *k* is a positive constant.

The acceleration at any instant, is proportional to the negative of the displacement at that instant. When the amplitude attains its maximum positive value *A*, the acceleration reaches its maximum negative value of $\frac{kA}{m}$, and when the body passes its equilibrium position ($x = 0$), at this point the acceleration is zero. The work done by the elastic restoring force can be represented in terms of the potential energy $U = \frac{1}{2} kx^2$ and the kinetic energy $K = \frac{1}{2} mv^2$.

According to the Principle of Conservation of energy,

Total energy $E = U + K$ is a constant.

That is $E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$ is a constant.

Total energy is also related to the amplitude A of the motion. It is equal to $\frac{1}{2} kA^2$

$$\therefore E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2$$

$$\therefore v = \pm\sqrt{\frac{k}{m}\left(A^2 \quad x^2\right)}$$

or

$$v = \pm w\sqrt{\left(A^2 \quad x^2\right)}, \quad \text{where} \quad w = \sqrt{\frac{k}{m}}$$

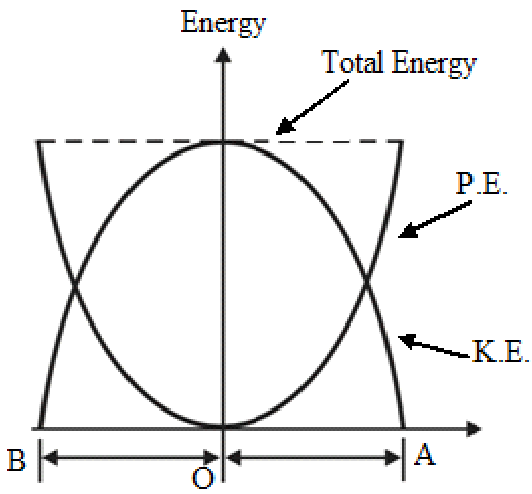


Figure 1.8 -The kinetic and potential energy variation of SHM

This relation gives the velocity of the body for any given position. An important characteristic of simple harmonic motion is that the frequency is not dependent on the amplitide of the motion. The kinetic and potential energy variation of SHM is given in Figure 1.8.

Period of oscillation of a mass on a spring

Consider vertical oscillations of a mass m suspended by a spiral spring. If the spring obeys Hooke's law, the extension caused by a mass m attached to it is directly proportional to the tension on the spring. The downward force mg, acts on the mass m is balanced by the spring tension, which causes to stretch the string by length l, as shown in the Figure 1.9.

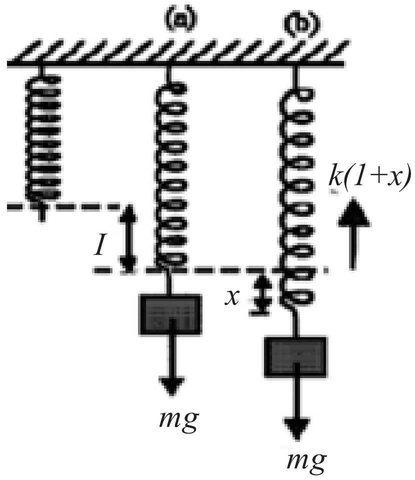


Figure 1.9 -Vertical oscillations of a mass suspended by a spring.

suspended by a spring.

$$mg = kl$$

where k is spring constant.

tension that acts downwards is $k(l+x)$. Then the new resulting restoring force that acts on the mass,

$$F = k(l + x) - mg$$

$$F = kl + kx - kl = kx \qquad (mg = kl)$$

When the mass m is released, it oscillates vertically moving upwards and downwards. Supposing it acquires an acceleration a, then at the instant its extension is x, by Newton's second law $-kx = ma$

$$a = -\frac{k}{m}x = -\omega^2 x$$

This relation implies simple harmonic motion of the mass.

The period of oscillation is given by,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

On squaring both sides,

$$T^2 = \frac{4\pi^2 m}{k}$$

In conducting an experiment the mass m that is attached to the spring can be varied and the corresponding period can be found. Using the ensuing results if a graph T^2 against m is plotted, it is a straight line but it does not pass through the origin as we may expect from the above equation. This is due to the mass of the spring, which has been ignored in our calculations. The effective mass of the spring can be found as detailed below.

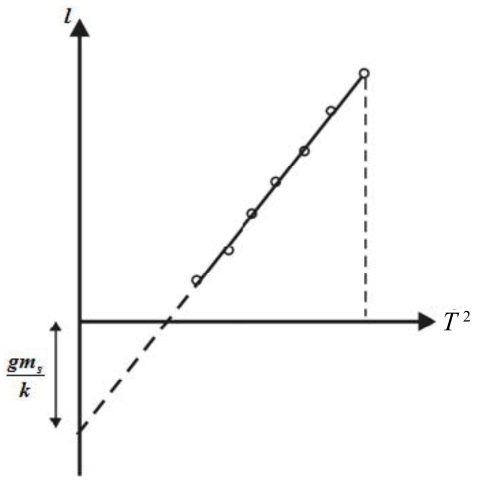


Figure 3.10 -The graph of l *vs* T^2

$$T = 2\pi\sqrt{\frac{(m + m_s)}{k}}$$

But, $mg = kl$

If m_s is the effective mass of the spring, the above equation can be modified as follows.

$$T^2 = \frac{4\pi^2}{k}\left(\frac{kl}{g} + m_s\right)$$

$$l = \frac{g}{4\pi^2}T^2 - \frac{gm_s}{k}$$

This graph can be obtained by measuring the extension of the spring l, and the corresponding time period T, for several differet masses m.

This would be a straight line graph of slope $\frac{g}{4\pi^2}$ and intercept $\frac{gm_s}{k}$ on the l axis.

Thus, g and m_s can be found. Theory suggests that the effective mass of a spring is about one third its actual mass.

Oscillations of mass-spring systems (springs in series)

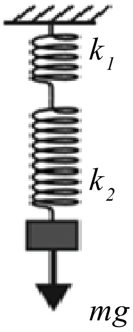


Figure 1.11- Springs in series

Two springs S_1 and S_2 whose force constants are k_1 and k_2 respectively, are connected in series as shown in the figure. A mass m is attached to one end of the spring system and the ensuing individual extensions of the springs S_1 and S_2 are x_1 and x_2 respectively.

As such, the total extension of the system,

$$x = x_1 + x_2$$

They both however, have the same tension. Therefore,

$$mg = k_1x_1 = k_2x_2$$

$$x_1 = \frac{mg}{k_1} \; and \; x_2 = \frac{mg}{k_2}$$

Now suppose k is the force constant for the entire combination of springs

$$\frac{mg}{k} = \frac{mg}{k_1} + \frac{mg}{k_2} \qquad \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

The period of oscilltion of the entire system is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

If the two springs are identical, $k_1 = k_2 = k$

Then $T = 2\pi\sqrt{\frac{2m}{k}}$

where k is the force constant of each spring.

Oscillations of a liquid in a U-tube

Consider a U tube T, with some liquid in it. If the liquid on one side of the tube is deppressed by blowing gently down that side as shown in the figure, the levels of the liquid in T, will oscillate for a short time about their respective initial positions O and C before finally coming to rest.

At a certain instant of time, suppose the liquid level on the left hand side, is at D, at a height x above the original position O. The level B of the liquid on the other side of the tube T, will thus be at a depth x below its original position C.

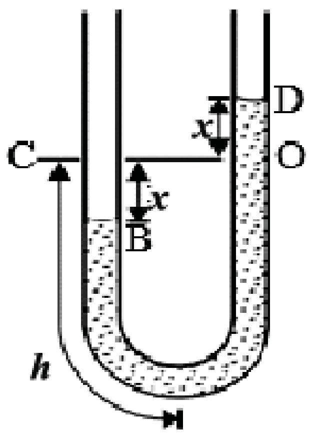


Figure1.12 - Oscillation of a liquid column

So the total excess pressure on the liquid is

excess height $\times$ liquid density $\times g = 2x\rho g$

force on liquid = pressure $\times$ cross sectional area

$= 2x\rho gA$

It is assumed that the area of cross sectional area the tube A is uniform.

The mass of the liquid in the tube = volume $\times$ density

$= 2hA\rho$

From Newton's second law,

$F = ma,$

$-2x\,\rho\, g\, A \quad = \quad 2hA\,\rho\, a$

the acceleration 'a' towards O or C is given by,

$$a = -\frac{g}{h}x = -\omega^2 x, \text{where } \left(\omega^2 = \frac{g}{h}\right)$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{h}{g}}$$

Damped oscillations

The amplitude of the oscillation of a pendulum gradually decreases to zero as a result of the resistive forces that arises due to air and the friction at the support. The motion of such a pendulum is therefore not equivalent to that of a perfect simple harmonic oscillator and is said to be **damped harmonic motion.**

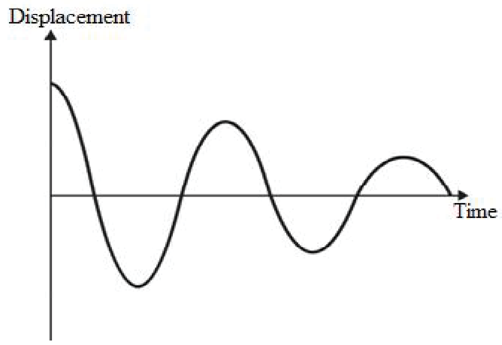


Figure 1.13(*a*) -Damped oscillations

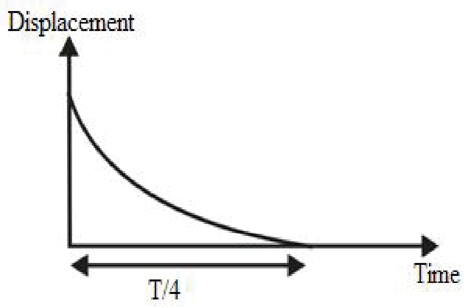


Figure 1.13 (b) Critically damped oscillations

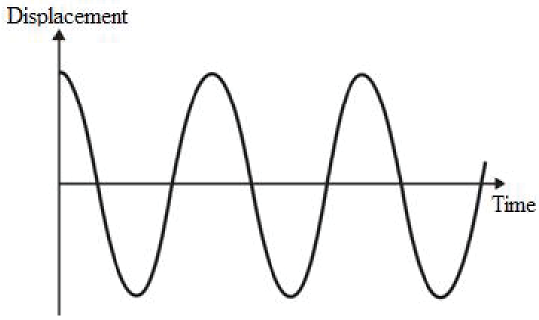


Figure 1.13 (c) - Free oscillations

The damping of the pendulum bob is greater when its oscillating in a liquid than in air. When the time taken for the displacement to become zero, is a minimum the system is said to be **critically damped**. Undamped oscillations are said to be **free oscillations** and their amplitude is constant as given in Figure 1.13 (c).

Very good example of damped oscillation is the action of shock-absorbers of a car, which critically damp the suspension of the vehicle and resist the setting up of vibrations. Otherwise it could easily make the control of the vehicle very difficult causing even damage to it. In the shock absorber the up-and-down motion of the suspension is opposed by viscous forces produced by the liquid passing through the transfer tube from one side of the piston to the other as shown in the Figure 1.14. Many systems are artifically damped to restrict unwanted vibrations.

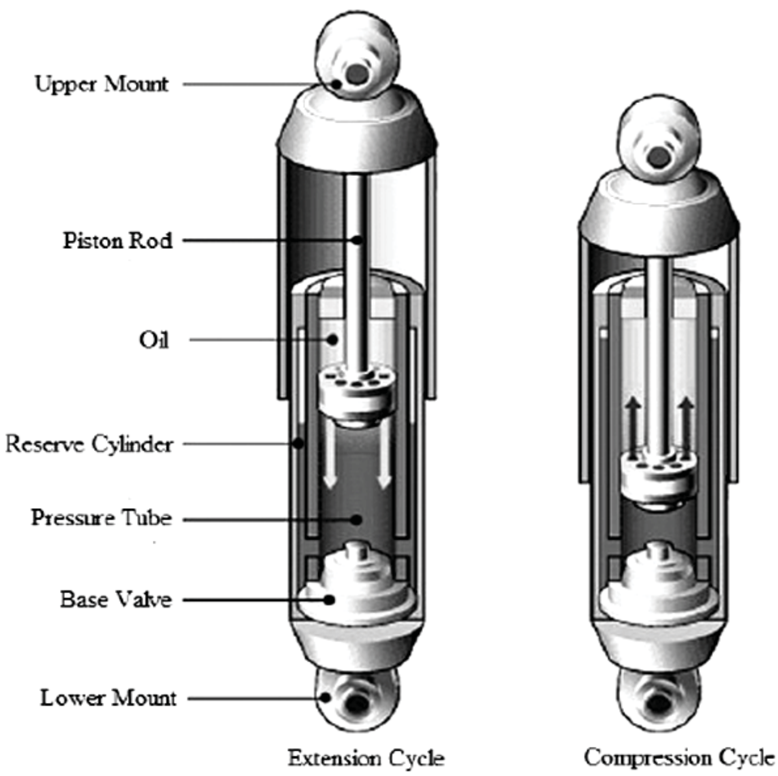


Figure 1.14 - Action of a shock

Forced vibrations

If the vibrations of a system is caused by an external periodic force, such vibrations are called forced vibrations. The wound spring of watch supply the necessary force require to maintain the function whilst in electrical watches the battery, and piezo vibrator do the process.

Demonstration of forced vibrations using Barton's Pendulum

In addition to twisting, XY swings in and out of the paper and therefore the lengths of the pendulums are measured from the beam.

The assembly illustrated in the Figure 1.15 given below comprises a number of pendulums P, Q, R and S, carrying masses much lighter than D, but still large enough for their motions to be considered undamped.

Suppose that the heavy pendulum D is slightly displaced so that it oscillates at its natural frequency in a plane, which is perpendicular to the paper. Its vibrations are transferred through the support string (XY), to the other pendulums and they start to oscillate. Since these pendulums are being forced to oscillate by D, they are said to be excuting forced oscillations. Once the motion has settled down observations show that all these pendulums vibrate with the same frequency (or at the natural frequency of D). The natural frequency of pendulum R, is equal to the forcing frequency, because it has the same length as D. It therefore, oscillates with a greater amplitude than P, Q and S. R is said to be resonating with D. The motion of R is a quarter of a period behind that of D. The shorter pendulums P and Q are nearly in phase with D, while the pendulum S is almost half a period behind D.

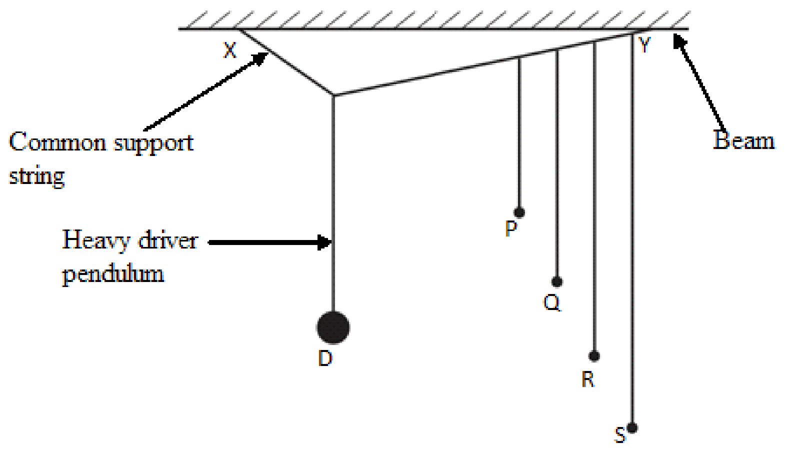


Figure 1.15- Apparatus to investigate forced oscillations Barton's pendulum

Resonance

Resonance occurs when the natural frequency of vibration of an object is equal to the external driving frequency, giving a maximum amplitude of vibration. Soldiers need to break step when crossing bridges. Failure to do so caused the loss of over two hundred infantry men in 1850. A spectacular example of resonance that is often quoted is the failure of the first suspension bridge over the Tacoma Narrows in USA. Wind caused the bridge to oscillate and one day strong wind set up twisting vibrations. The resulting amplitude of these vibrations increased due to resonance until eventually the bridge collapased [see Figure 1.16(*a*)]. Another example is the build up of amplitude of ascillations on a spring board as a result of a diver bouncing on it at its natural frequency. It has also been reported that an opera singer was able to shatter a wine glass by forcing it to vibrate at its natural frequency [see Figure 1.16(*b*)]. Tunning of a radio channel on a television receiver is a very good example of electrical resonance.

Figure 1.16(*a*) - Collapsing of Tocoma Narrrows Bridge

Figure 3.1.17(*b*) - Shattering of a wine glass

Chapter - 02

Wave motion

Next, our focus is to understand the nature of mechanical waves which are transmitted by oscillations of particles in a medium. The objective of this section is to introduce some general properties of waves. Under this topic we consider about different properties of waves. Another major consideration is a useful phenomenon called Doppler effect. Electromagnetic waves are studied as it has so many applications in modern world. Another application in modern world is LASER which we discuss here.

Waves can be classified as being eiher mechanical or electromagnetic. Mechanical waves (example:- water waves, sound waves, waves in a stretched string) require a material medium for their propagation. Disturbances due to a vibrating source produces to-and-fro vibrations of particles in the medium. As a result mechanical waves are formed. Electromagnetic waves (eg. light, radio, X rays, etc.,) can travel through a vacuum and their progress is impeded to some extent by the presence of matter. All types of electromagnetic waves travel with the same speed in free space.

By throwing a stone in to a pond, water waves can be formed on the surface of water. A floating object on the surface of water can be observed to bob up and down as ripples flow past it on the water surface, but the floating object does not move along with the waves. The water particles in the path of the ripples are subject to vertical up and down motion. The resulting disturbance causes the water particles to move in simple harmonic motion.

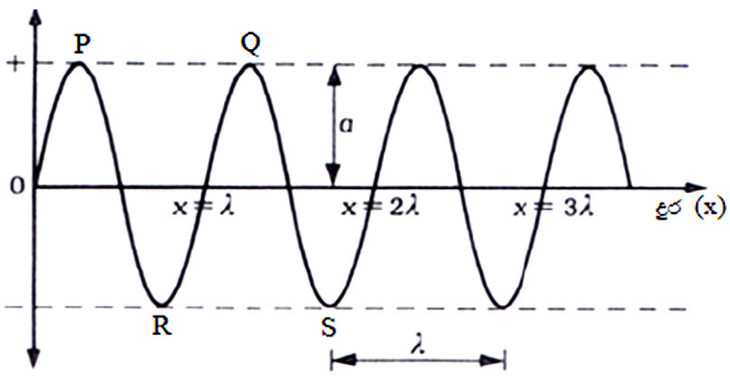


Figure 2.1- Snap shot of a travelling wave along a string

Suppose we take a snap shot of waves travelling along a string, the photograph would resemble the above Figure 2.1. It shows the position of the string at the instant the snapshot was taken and it has a shape of a sinusoidal wave.

Therefore, the displacement y of each particle of the string can be represented by sinosidal form, $A\sin\omega t$, where A is the amplitude and ω is angular frequency ($2\pi f$ or $\frac{2\pi}{T}$) of the wave (see Figure 2.1). The points P and Q on the figure are called crests, while the points R and S are called troughs.

$$y = A\sin 2\pi ft = A\sin 2\pi \frac{v}{\lambda} t$$

If the parameter (coordinate) x represents the distance measured from the fixed point (origin) O, along the wave and λ is the wavelength of the wave,

$$y = A\sin\frac{2\pi x}{\lambda} = A\sin kx, \text{where } k = \frac{2\pi}{\lambda}$$

The quantity k is called the wave number.

Here the wave is travelling at speed v along the $+x$ direction. The equation gives the displacement y at the distance x from the origin O, and at time t (see Figure 3.18).

Wave front

A **wave front** is a line or surface, in the path of a wave motion, on which the disturbances at every point have the same phase.

Progressive waves

When the wave front moves through the medium is called a progressive wave. There are two types of progressive waves called transverse and longitudinal. Snap shot given in Figure 2.1 does not show which direction the waves are travelling in. But the successive snap shots as in Figure 2.2(a) are useful to identify the wave travelling direction. Further to that, Figure 2.2(c) is the superimposed snap shot of the two waves given in Figure 2.2(b). It clearly shows the rightward shift of the second wave.

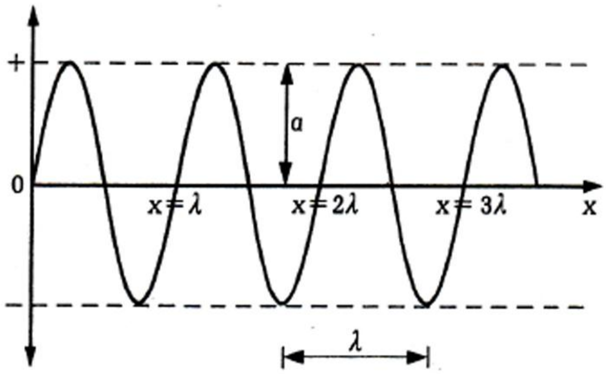


Figure 2.2(*a*) Successive snapshots of a progressive wave

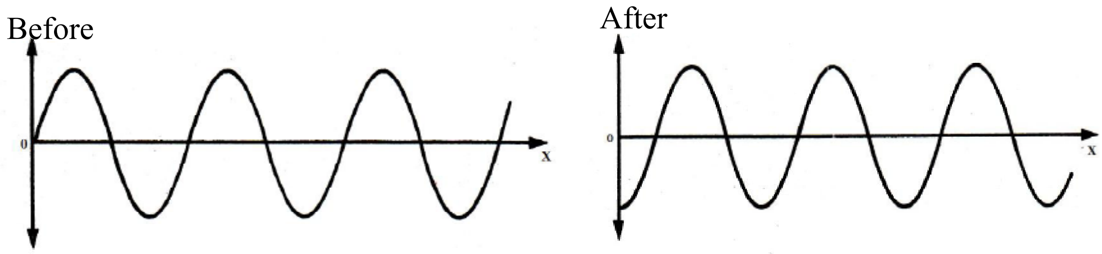


Figure 2.2(*b*) Successive snapshots of a progressive wave

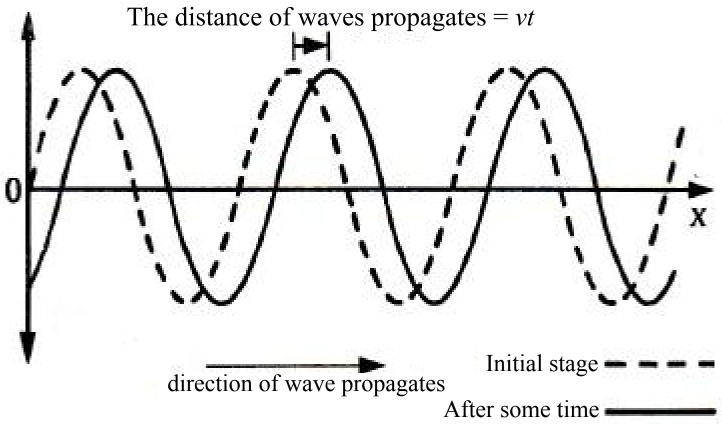


Figure 2.2(*c*) Superimposed snap shots

The equation for the distance of the wave indicate in dotted line is, $y = A\sin\left(\frac{2\pi x}{\lambda}\right)$

The wave propagation for the positive side after time 't' is indicated in graph 2.2 (b). To find the displacement of the wave, the equation mention in below can be used.

$$y = A\sin\frac{2\pi}{\lambda}(x - vt)$$

This is the common equation for the waves travel to the positive direction with 'v' velocity, 'A' amplitude and 'λ' wavelength.

Transverse waves

Waves produced on the surface of water due to wind or by throwing a stone into a pond is a very familiar sight for us. We have noticed the circular ripples, which spread out from the spot where the stone entered the water. These ripples are an example of rhythmic wave motion travelling as circular wavefronts. In this, the surface of water (particles of water) is displaced vertically as indicated by arrows in the Figure 2.3. The amplitude A, and the wavelength λ, of transverse water waves are also shown in the same figure. The distance, which the water surface (particles of water) displaced is given by vertical arrows. The highest point of water surface (medium) raised is called crest and the lowest point of water surface (medium) sinks is called trough. Crests and troughs thus formed travel horizontally, or in other words, along a direction perpendicular to the direction of motion of the surface of water. Thus there are two types of motions, the first comprise the particles of the medium and the second is the motion of the wave. As such, water waves formed are transverse waves and so are most of the waves that we come across in day-to-day life.

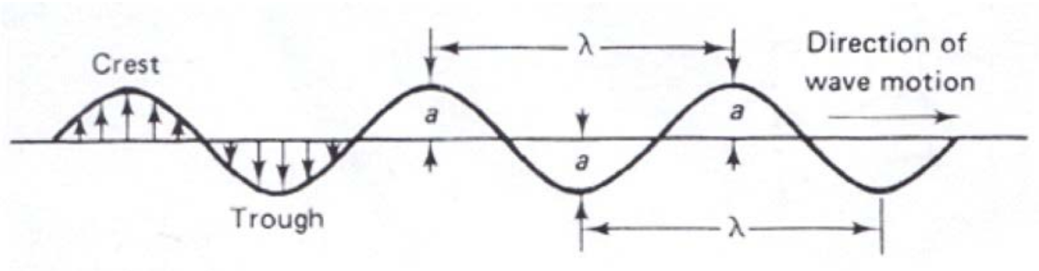


Figure 2.3 -Displacements of particles of water waves

Somewhat a simpler type of wave can be observed when one end of a piece of string is moved up and down in a direction perpendicular to its length. The particles of the string near the end, exert a drag on their neighbours so that these particles begin to oscillate. The next particular particle begins to oscillate up and down slightly later than the one immediately before it. This process continue throughout the string. The net result is that the string presents the appearance of a series of equidistant crests and troughs, which travels forward with a certain velocity, called the wave velocity. The direction of arrows in Figure 2.3 shows the velocities of the particles. In the Figure 2.4 , you can find the pairs of particles that are in phase. The distance between two successive in-phase particles such as 3 to 11 is called a wavelength. The distance between particles 2 to 10 is also equal to one wavelength. There are particles such as 2 and 6 which have exactly opposite displacement and opposite velocities at every instant. They are said to be exactly out of phase.

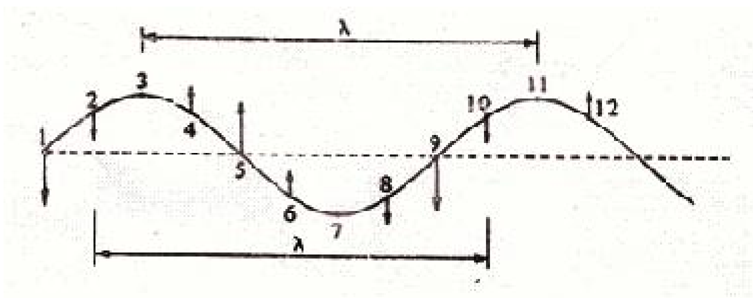


Figure 2.4- Velocities of particles of string waves

Longitudinal waves

If the vibrations are along the direction of the travel of the wave, they are called longitudinal waves. Thus the particles of the medium oscillate along the direction of motion of the wave. Sound waves and compression waves in springs are longitudinal. Figure 2.5 illustrates the type of waves in a slinky supported by strings at various intervals along its length. If one end of the slinky is displaced sharply backwards and forwards along its length, a compression and then a rarefaction can be seen moving along the spring. Due mainly to the longitudinal motion of the wave particles, sound waves consists of a series of compressions followed by rarefactions.

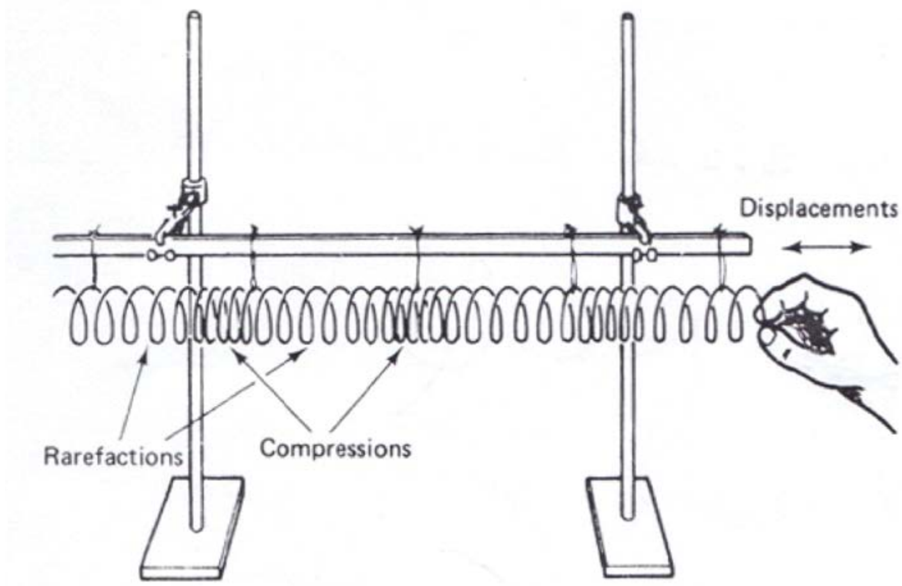


Figure 2.5 Compression waves in a slinky

Figure 2.6 shows how a vibrating tuning fork sends out a sound wave. When the prong moves to the right it compresses the air particles and bring them close together. This disturbance is then transmitted from particle to particle through air, resulting in the formation of a pulse of compression that moves outwards. Similarly a reverse movement of the prong gives rise to a pulse of rarefaction in air moving in the same direction.

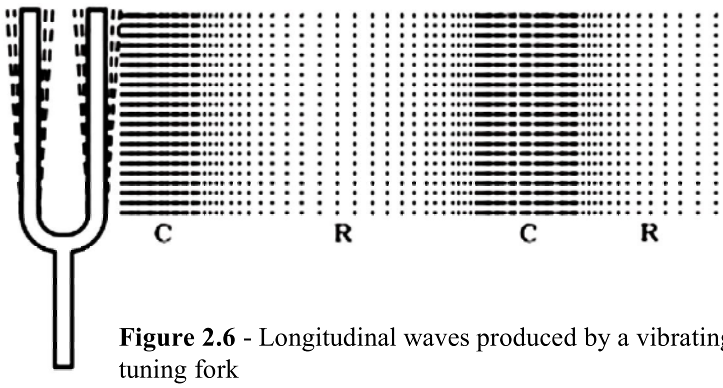


Figure 2.6 - Longitudinal waves produced by a vibrating tuning fork

It is important to note that the particle at the centre of a compression is moving through its rest position in the same direction as the wave, while the particle at the centre of a rarefaction is moving through its rest position in the opposite direction as that of the wave. As in the case of a transverse wave the distance between two successive particles in the same phase is called the wavelength, and the same relationships applies when carrying out calculations.

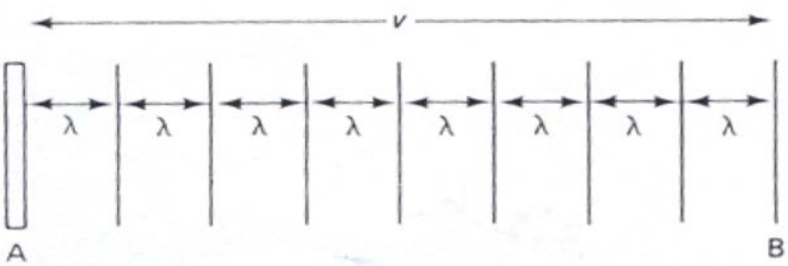


Figure 2.7 - The straight wave of frequency 8 Hz are emitted at A, and they fill the space AB after 1 s

The relationship among the wave velocity *v,* frequency *f* and the wavelength *λ,* can be written in the form of $v = f\lambda$. This equation can be applied to all types of waves.

Consider the example depicted in the Figure 2.7, where a source of straight waves, A, emits eight cycles in one second. Their frequency is therefore, 8 Hz. Now suppose the wave velocity is *v*, and the first wave has reached the position B, after 1s, then AB = *v*. As such, all eight wavelengths are within in the distance AB. So the wavelength is given by $\lambda = \frac{v}{8}$. If the frequncy had been 16 Hz then all sixteen wavelengths would have been contained in *v* after 1s, and their wavelength would have been $\frac{v}{16}$.

Worked Example:-

Measurement of the frequency and the wavelength of waves enable their velocity to be calculated. For example, we can find the speed of radio waves in air from the knowledge that the transmission frequency 200 kHz has a wavelength of 1500 m.

$$v = f \times \lambda$$

$$v = 200 \times 10^3 \times 1500 = 3 \times 10^8 \text{ m s}^{-1}$$

Sound waves travel much more slowly in air than electromagnetic waves. That's why the light of a lightning flash comes sooner than the sound of its. It can be calculated that, if the time difference between the sound and the light of a ligthning is 3 s, the distance to the lightning is 1 km from the observer.

Chapter - 03

Properties of waves

All known wave motions share four common properties. They can be reflected, refracted, diffracted and also exhibited interference. We shall now describe some experiments concerning water waves, which show that in some way their behaviour bears a striking resemblance to that of light.

The motion of water waves could be studied by looking at the ripples formed in a ripple tank. It comprises a shallow transparent tray of water with a point light source above it and a white screen on the floor below it.

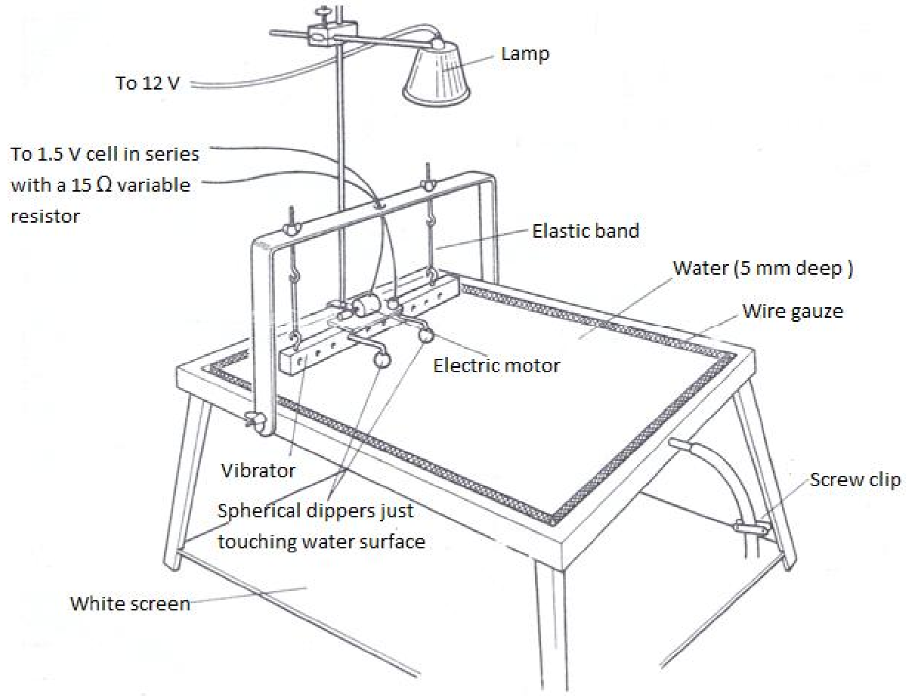


Figure 3.1 - Demonstration of propagation of water waves using the tank

Before adding water the tray is levelled with a spirit-level to ensure a uniform water level. Water is added until the depth of water to be about 5 mm. Straight parallel waves may be produced by a horizontal metal strip, or circular waves by a vertical ball ended rod. When either of these is dipped into water, a pulse of ripples is sent across the surface. Alternatively, continuous ripples may be obtained by fixing the dipper to a horizontal bar suspended by rubber bands. The bar is moved up and down by the vibrations of a small electric motor having an eccentric metal disc on its rotating spindle. A rheostat in the motor circuit controls the speed and hence, the frequency of the waves sent out. Owing to the lens effect of the wave crests and troughs, the light source produces a bright and a dark wave pattern on the white screen. There white bands represent crests and dark bands represent troughs.

Reflection of Waves

The Figure 3.2 shows how a plane wave rebound from a straight barrier where the incident angle is 45 ^{o}. The waves retain their speed and wavelength as they remain in the same medium. Their directions of motion before and after reflection make equal angles with the normal, which is drawn at 90 ^{o} to the barrier.

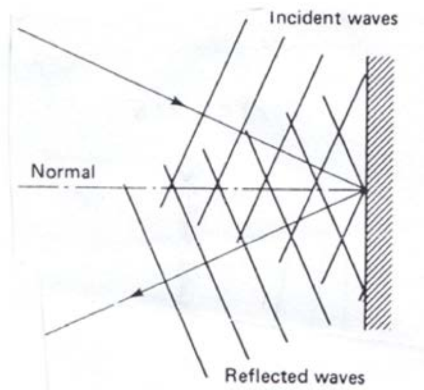


Figure 3.2(a) Reflection pattern obtained with plane waves

Here again their wavelength and speed are unchanged, but the curvature of the waves is altered so that the reflected waves are centred at S', which is the image position of the source behind the barrier. It is obvious where an object placed in front of a plane mirror gives rise to a virtual image situated at the same distance behind the mirror.

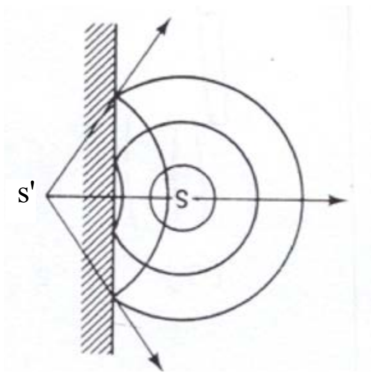


Figure - 3.2*(b)* Reflection pattern obtained with circular waves

Refraction of Waves

In passing from one medium to another, waves undergo changes of speed and wavelength, but their frequency remains unchanged. Figure 3.3 shows a special instance of this when straight ripples enter a region of shallow water. Changes of speed and wavelength occur at the boundaries between shallow and deep regions but waves continue to move in the same dirction as before. This can be shown by placing a rectangular piece of glass of suitable thickness in the tank to reduce the depth of water. Waves travelling along the normal to the boundary are not deviated when they undergo changes of speed and wavelength. But waves crossing the boundary obliquely are deviated and travel in a new direction. In other words, refraction occurs.

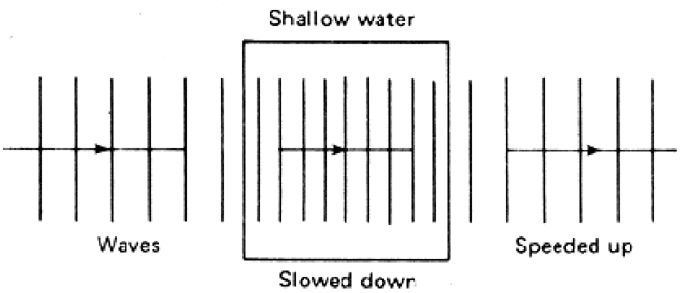


Figure 3.3- Changes of wave parameters in refraction

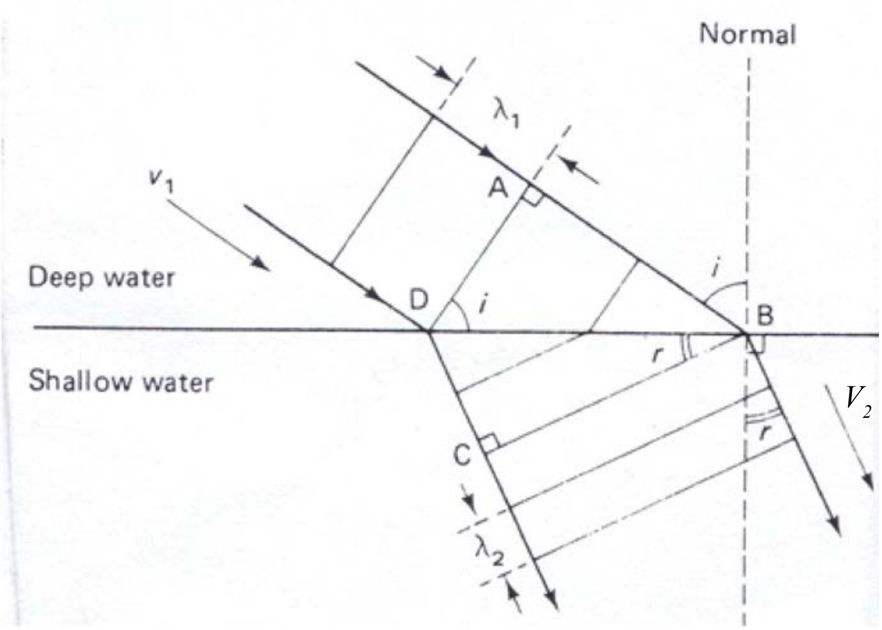


Figure 3.4 - Changes of wave parameters in refraction

As shown in Figure 3.4, the wavefront AD in deep water becomes BC in shallow water. The distances AB and DC are travelled in both media in equal time, and their ratio is the ratio of two wave speeds v_1 and v_2. As AB is greater than DC we can say that $v_1 > v_2$.

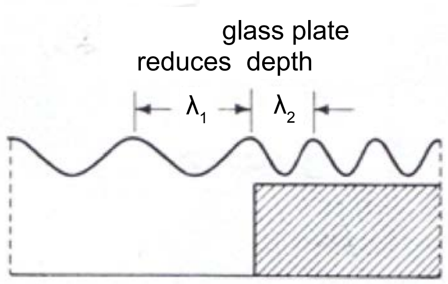


Figure 3.5 - Decreasing of wavelength in shallow water

The equation $v = f\lambda$ applies to the waves on each side of the boundary BD, where they have the same frequency, and from this it follows that their wavelengths are at the same ratio as their velocities.

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{AB}{DC}$$

The above ratio can also be written as

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{AB}{BC} \times \frac{BD}{DC}$$

From the right angled triangles ACD and CBD (in Figure 3.4), the ratio

$$\frac{AB}{BC} \times \frac{BD}{DC} = \frac{\sin i}{\sin r}$$

where i and r are angle of incidence and angle of refraction respectively.

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{\sin i}{\sin r} = n \text{ (a constant)}$$

where the constant n is called the refractive index for water waves, passing from deep to shallow waters and v_1, v_2 are wave velocities in deep water and shallow water respectively.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{400} - \frac{1}{\infty} = \frac{1}{f} \Rightarrow f = 400 \text{ cm}$$

This is Snell's law of refraction and it holds for electro-magnetic, sound and water waves.

Diffraction of Waves

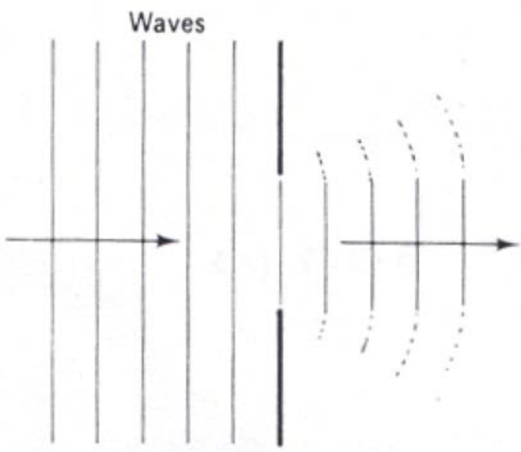


Figure 3.6 (a) Effect of a wide gap

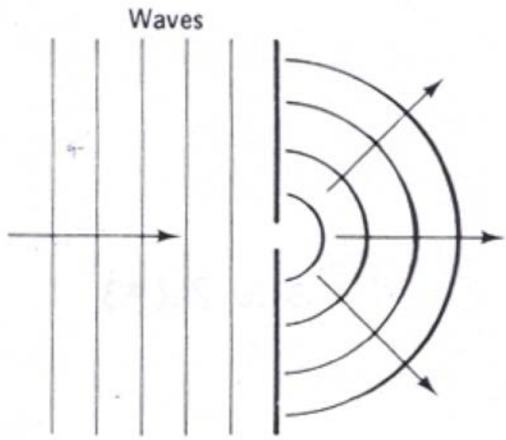


Figure 3.6 (b) Effect of a narrow gap

Figure 3.6 (a) and Figure 3.6 (b) show the passage of waves through gaps in a barrier. Waves in a ripple tank passing through the centre of a gap continue straight on, but there is a tendency for the end of each wavefront to deviate into the geometrical shadow area behind the barrier. As the waves remain in the same medium there is no change in their speed or wavelength. In Figure 3.6 (a) where the gap is much wider than the wavelength of the waves the effect is not very pronounced.

In contrast, it is much noticeable in Figure 3.6(b), where the width of the gap and the wavelength of the waves are comparable. In general, we can say that the diffraction that takes place when apertures are wide compared to the wavelengths of the waves are of much low significance.

Sound waves also diffract while passing around corners and obstacles. Conversations that take place in a corridor can be heard in a room if its door is kept open though those taking part in the conversation cannot be seen. The sound waves reaching the doorway diffract into the room although light waves of much shorter wavelegth do not diffract.

Principle of Superposition

When two or more waves traverse in same medium the resultant displacement of any particle of the medium is the sum of the displacements that the individual waves would produce.

Coherent sources

Coherent sources are those which maintain waves of constant phase difference, which means that they must have the same frequency. In practice coherent sources are derived from a single source.

Interference of waves

Interference occurs when two or more similar sets of waves are superimposed on each other. Such wave trains are obtained from coherent sources. The result of two wavecrests arriving simultaneously at a point is a large crest, and two troughs arriving together result in a large trough. This is called constructive interference and the waves are said to be 'in phase'. When a crest and a trough of similar amplitude arrives at a point simultaaneously, the effect is to produce a resultant displacement of zero amplitude. This is called destructive interference and the waves are said to be 'out of phase'. Complete cancellation to occur the amplitudes of the superposing waves must be equal.

Interference of water waves

These create an interference pattern in a ripple tank equipped with two small circular dippers and an electrical vibrator. Figure 3.7 illustrates instantaneous positions of the waves emitted by two identical sources and it shows the paths of constructive and destructive interferences. Whenever, constructive interference occurs these waves have double the amplitude of waves from one source alone.
The rate of energy flow along these paths is therefore, greater than for one source alone. In constrast,

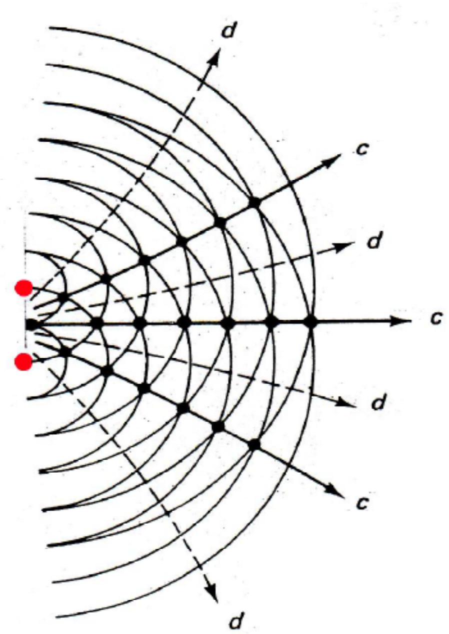


Figure 3.7 Interference of water waves

whenever destructive interference occurs, the wave amplitude tends to zero and there is no flow of energy along these paths. Figure 3.7 shows an interference pattern of waves emerging in phase from two identical slit sources. Paths marked as 'c' indicate constructive interference and those marked as 'd', indicate destructive interference.

Stationary waves

We can explain the formation of stationary waves using the principle of superposition. Figure 3.8 represents two progressive waves of equal amplitude and frequency travelling in opposite directions. In figure the wave drawn in broken lines is travelling from left to right and the wave drawn in dotted lines is travelling from right to left. The graph on the top shows the waves at an instant, at which they are in phase. Superposed wave is illustrated by a solid line in same figure, which has twice the amplitude of either of the two progressive waves. The second graph represents the situation a quarter of a cycle later, when the the two progressive waves each has moved a quarter of a wavelength in opposite directions.

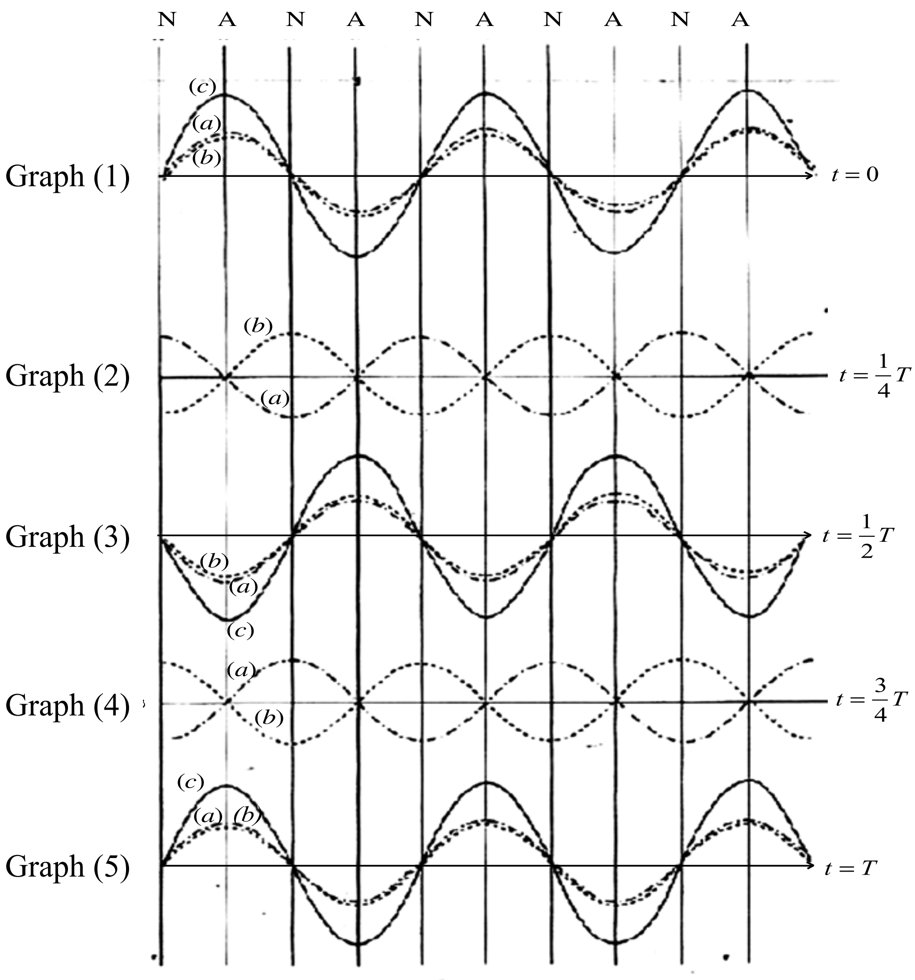


Figure 3.8 - Formation of stationary waves by superposition of two progressive waves travelling in opposite directions

In the third graph half a period from the start, waves are again in phase, with maximum displacemnet for the resultant. The process continues through the forth graph, showing the next out of phase situations, with zero displacement of the resultant every wave. Finally the fifth graph, one period on from the first, brings the two waves into phase again.

The dotted and broken lines are the displacement-distance graph of two such waves at successive equal time intervals. The solid line in each case is there resultant at these instances, formed by superposition. The formation of stationary loops can now be understood.

Note that,

i. Points such as N, called nodes of the stationary waves, where the displacement is always zero.
ii. Within one loop all particles oscillate inphase but with different amplitudes. So all points (except nodes) have their maximum displacements simultaneously. Points such as A with the greatest amplitude are called antinodes.
iii. Oscillations in one loop are in antiphase with those in an adjasent loop.
iv. The wave length of a stationary wave is twice the distance between successive nodes or successive antinodes and equals the wavelength of either of the progressive waves. Large amounts of energy are stored locally in stationary waves and become trapped with the waves, there is no transmission as with progressive waves. This is an important difference between stationary and progressive waves.

Application of stationary waves

The notes we hear from a voilin are created by the vibrations of its strings. The wave patterns on the vibrating string are stationary waves. The waves in the air which carry the sound to our ears, transfer energy and are therefore called progressive waves.

Beats

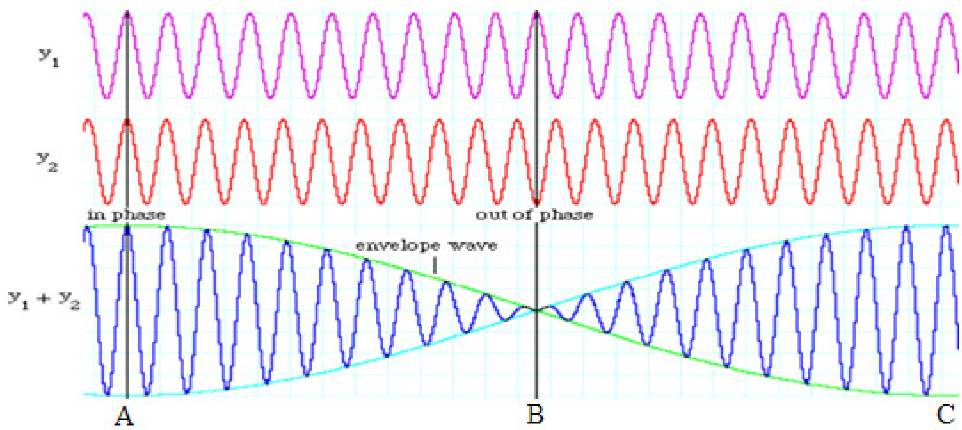


Figure 3.9- Formation of beats

When two notes of slightly different frequencies but similar amplitudes are sounded together, periodically increasing and decreasing loudneses are produced. These sounds are called beats. The production of beats is a wave effect explained by the principle of superpostion. The When two notes of slightly different frequencies but similar amplitudes are sounded together, periodically increasing and decreasing loudneses are produced. These sounds are called beats. The production of beats is a wave effect explained by the principle of superpostion. The displacement time graphs for the wave-trains from two sources (tuning folks) of nearly equal frequencies are shown in Figure 3.9.

At the instant such as A the waves from two sources arrive in phase and reinforce to produce a loud sound. The phase difference then increases until a compression or rarefaction from one source arrives at the same time as a rare faction or a compression from the other. The observer hears a little or nothing at B. But later waves are again in phase at point C, and a loud sound is heard.

The periodic increase and decrease in amplitude is a result of successive occurences of constructive and destructive interference between the two notes as they repeatedly become in phase and then out of phase with each other. The number of times the sound reaches maximum in one second is called beat frequency, $f_b = | f_1 - f_2 |$ where f_1 and f_2 are the frequencies of two tuning forks.

Demonstration of beats using an oscilloscope

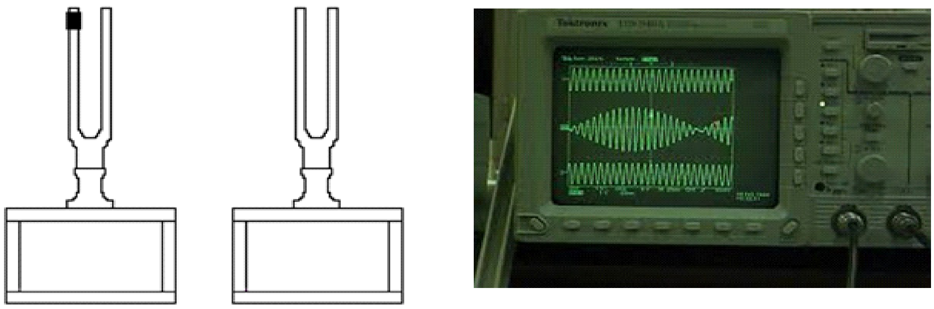


Figure 3.10

Take two tuning forks with same frequency and attach a small mass of plasticine to a prong of one of them, and mount them as shown in Figure 3.10. They are kept 50 cm apart and place the microphone in between them, You can hear beats and corresponding wave pattern can be displayed on oscilloscope screen through the microphones. The wave pattern displayed is somewhat simillar to Figure 3.9.

Uses of beats

The phenomenan of beats can be used to determine an unknown frequency if the other frequency is known.

Beats are also used to tune musical instruments to a given note. As the instrument note approaches the given note beats are heard. The instrument may be regarded as "tuned" when beats occur at very slow rate.

Polarization of transverse waves

Mechanical waves

Polarization of waves can be demonstrated simply by using a thin string and two narrow slits as illustrated in the Figures 3.11(a). If the waves generated in a string is passed through two slits B and C, it is obvious that, unless both slits are parallel to each other and the string vibration is in the same plane the emergent wave from the second slit is not possible. If slit D is rotated, the string vibrations do not pass through D until both slits become parallel to each other. Figure 3.11(b) shows the wave that has been stopped at C due to the crossed slits. The wave between BC and CD are plane polarized. The wave between AB is unpolarized where the string could be vibrated in any direction.

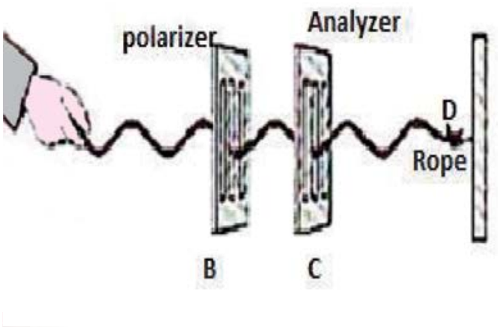


Figure 3.11 (a)

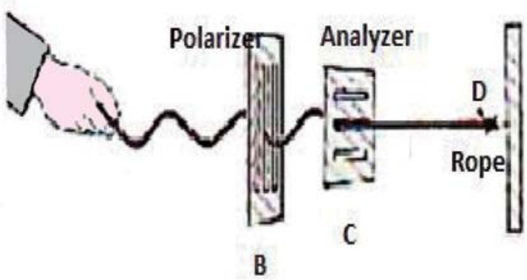


Figure 3.11 (b)

Figure 3.11 Formation of plane polarized waves.

Polarization of Light waves

Years ago it was discovered that certain natural crystals affected when light is passing through them. Tourmaline, quartz, calcite are some examples of such crystals. Suppose two tourmaline crystals P and Q are placed parallel (their axes *a* and *b*) to each other as in Figure 3.12 (c), (d).

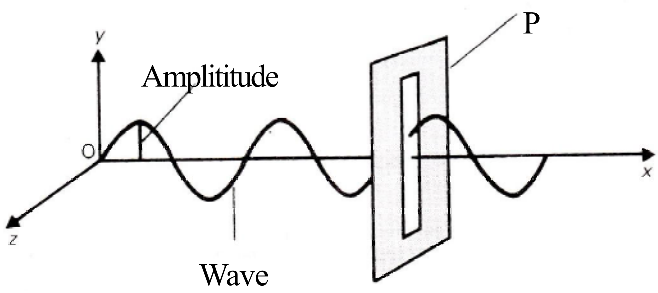


Figure 3.12 (c)- Formation of plane polarized light waves

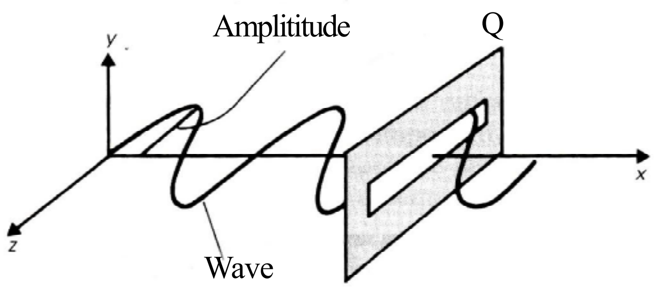


Figure 3.12 (d)- Formation of plane polarized light waves

Tourmaline is a crystal which, because of its molecular structure, transmits only those vibrations of light parallel to its axis.

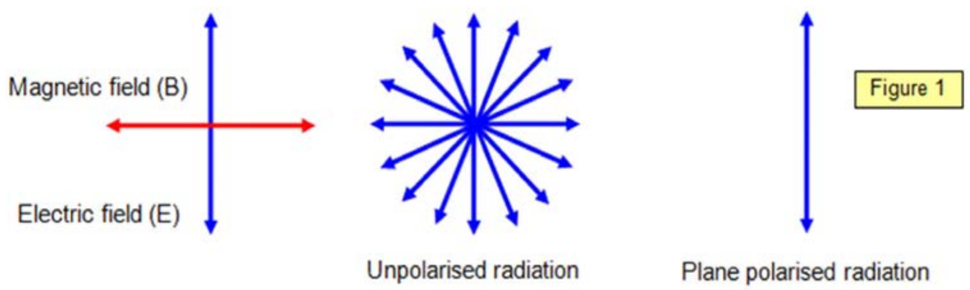


Figure 3.13

In ordinary light, the electric and the magnetic fields are vibrating in millions of planes perpendicular to the direction of light propagation. Amplitudes of all vibrations are equal. In sunglasses, Polaroid are used to cut certain vibrations of harmful UV rays those could damage our eye. Several methods are used to obtain plane polarized light. Reflection of light by a pile of glass plates and double refraction are two common mechanisms.

Chapter - 04

Stationary waves in stretched strings

Formation of stationary waves using principle of superposition has been already discussed. We have seen how a string attached at its ends vibrates making loops along it when subjected to a vibration. The transverse wave which propagates along the string as a result of the vibration gets reflected at its ends, thereby fulfilling conditions to form stationary waves. The wave travelling forward superpose with the reflected wave to form stationary waves. Since the ends of the string are rigidly fixed, rigid reflections occur at the two ends. It's known that in a rigid reflection a node is formed at the reflecting surface. Thus if the string can vibrate freely, all modes of stationary waves formed along it should have nodes at the two ends.

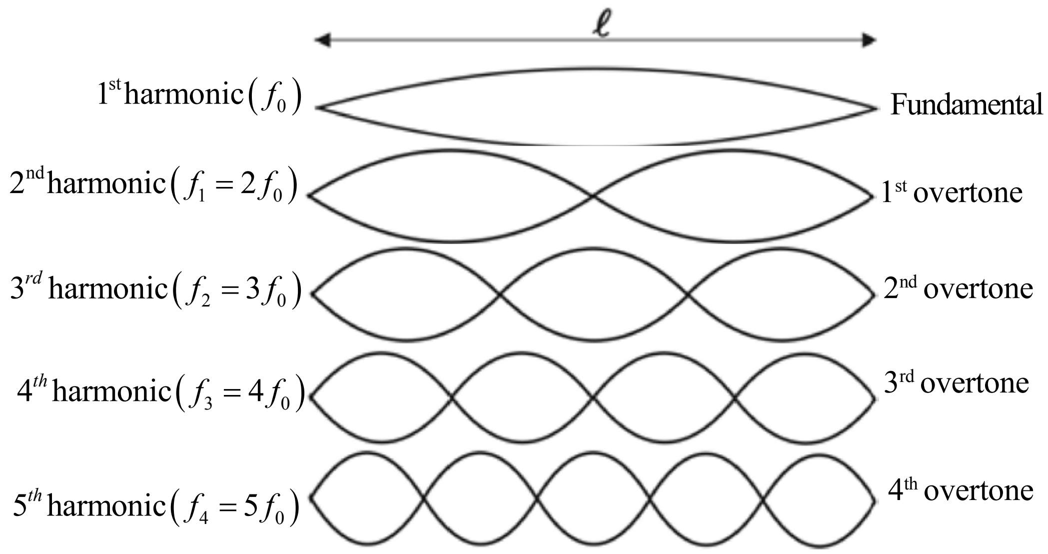


Figure 4.1- Vibration modes of a string

The first mode of vibration is called the fundamental. The modes which follow are called overtones. Figure 4.1 shows how they are named. Let f_0 be the frequency of the fundamental tone and l the length of the string.

Then we can see that the wave length of the fundamental stationary wave is $\lambda_0 = 2l$. By substituting in $v = f\lambda$. The frequency of the fundamental tone becomes $f_0 = \frac{v}{\lambda_0} = \frac{v}{2l}$ In the first overtone, since $\lambda_1 = l$, $f_1 = \frac{v}{\lambda_1} = \frac{v}{l}$

As this is twice $\frac{v}{2l}(= f_0)$ this frequency is $2f_0$. Accordingly the frequency of the 2^{nd} overtone is $f_2 = 3\left(\frac{v}{2l}\right) = 3f_0$ and that of the 3^{rd} overtone is $f_3 = 4f_0$. The frequencies of the overtones can be found conventionally in this manner.

It can be shown that all types of above stationary waves can be formed in a string by using a small speaker as the source of vibrations with an audio frequency generator as shown below.

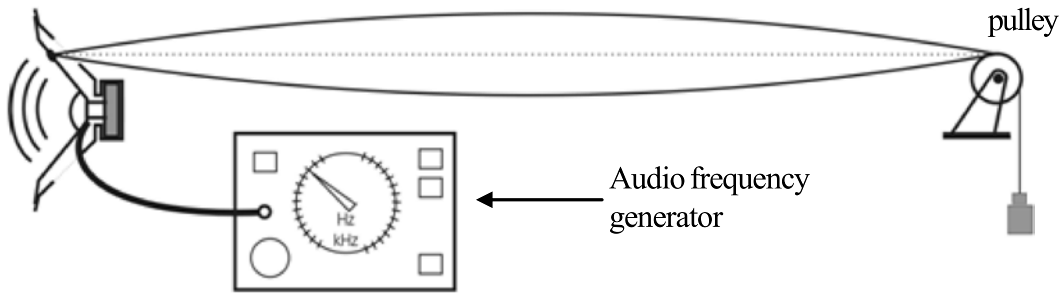


Figure 4.2

Paste one end of the string using a glue to the middle of the paper bridge of the speaker. Pass the other end of the string over the pulley and hang a load sufficient to keep the string stretched. Now connect the audio frequency generator to the speaker and starting from a low frequency increase the frequency of the generator until the first stationary wave form is produced along the string. If it is not possible, obtain that situation by changing the position of the pulley. Then, as shown in Fgure 4.3, a stationary wave consisting of one loop, which indicates the fundamental tone, will be seen on the string. Thereafter, by increasing the frequency (f_0) of the generator by its multiples such as $2\,f_0, 3\,f_0, 4\,f_0, \ldots$ .The wave forms of other frequencies such as those shown in Figure 4.2 will appear along the string.

Velocity of transverse waves along a string

If T is the tension in a string and m its linear density (mass per unit length) then the velocity v of a transverse wave along the string can be expressed as,

$$\boxed{v = \sqrt{\frac{T}{m}}} \quad \text{\_\_\_\_\_\_\_\_\_}(4.1)$$

Using the above expression and the wavelength of the wave, an expression for the frequency f can be obtained as follows.

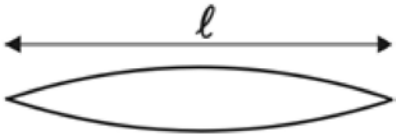


Figure 3.4.3

$$\sqrt{\frac{T}{m}} = f\lambda$$

$$\boxed{f = \frac{1}{\lambda}\sqrt{\frac{T}{m}}} \quad \text{\_\_\_\_\_\_\_\_\_}(4.2)$$

When the string vibrates in the fundamental tone (Figure 4.3). The wavelength λ_0 can be seen to be half its length .

$$\lambda_0 = 2l$$

Substituting in equation (4.2)

$$f_0 = \frac{1}{\lambda_0}\sqrt{\frac{T}{m}}$$

$$f_0 = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \text{\_\_\_\_\_\_\_\_\_(4.3)}$$

The above expression gives the frequency f_0 of the fundamental vibration mode of a freely vibrating string. Similar expressions can be obtained for the overtones of a freely vibrating string. Considering the state of the 1st overtone,

But since $l = \lambda_1$

$$f = \frac{1}{\lambda}\sqrt{\frac{T}{m}}$$

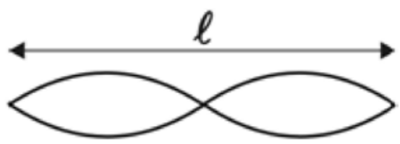


Figure 4.4

The value of f_1 in the 1st overtone can be seen to be,

$$f_1 = \frac{1}{l}\sqrt{\frac{T}{m}}$$

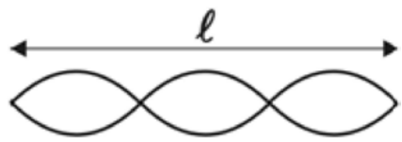


Figure 4.5

In the 2nd overtone, the length of the string,

$$l = \frac{3}{2}\lambda$$

$$\therefore \lambda = \frac{2}{3}l$$

Hence $f_2 = \frac{3}{2l}\sqrt{\frac{T}{m}}$

Thus it is seen that,

Frequency of the fundamental $f_0 = \frac{1}{2l}\sqrt{\frac{T}{m}}$

Frequency of the first overtone $f_1 = \frac{1}{l}\sqrt{\frac{T}{m}} = \frac{2}{2l}\sqrt{\frac{T}{m}} = 2f_0$

Frequency of the second overtone $f_2 = \frac{3}{2l}\sqrt{\frac{T}{m}} = 3.\frac{1}{2l}\sqrt{\frac{T}{m}} = 3f_0$

Hence since the frequency of the n^{th} overtone is nf_0,

$$f_n = \frac{(n+1)}{2l}\sqrt{\frac{T}{m}}$$

Similarly the 1st harmonic of a free string is its fundamental of frequency f_0 . The frequency of the 2nd harmonic will be $2f_0$ and that of the 3rd harmonic will be $3f_0$ and so on.

$\therefore$ Frequency of the nth harmonic $f_n = nf_0 = \frac{n}{2l}\sqrt{\frac{T}{m}}$

The sonometer

The sonometer is an apparatus in the form of a box made of wooden planks on which a stretched string, is attached as given in the Figure 4.6. The tension of the string can be varied by adding or removing mass placed on the weight hanger.

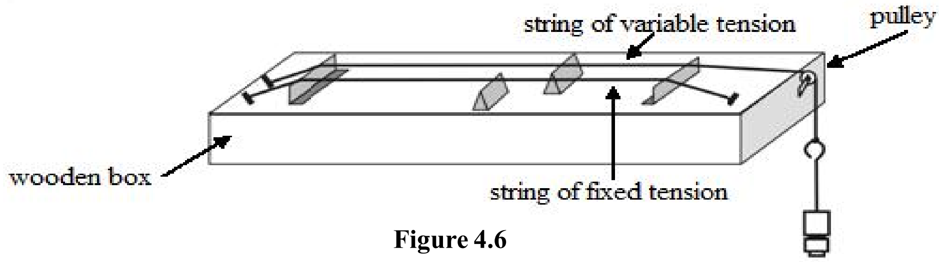


Figure 4.6

Determination of the frequency of a tuning fork using the sonometer

Vibrate the tuning fork and hold it with its handle touching the sonometer box. Now, place the peg to have the shortest vibrating length of the wire and adjust the peg gradually until the fundamental resonance situation is reached. This resonance state can be identified easily by placing a small paper rider on the middle of vibrating portion of wire. It is to be noted that the paper rider has to be always at the middle when adjusting the pegs.

The peg has to be adjusted until the rider is thrown off, when an antinode is formed in the middle of the wire. Now measure the length l_0 of the resonated portion of the wire using a metre ruler.

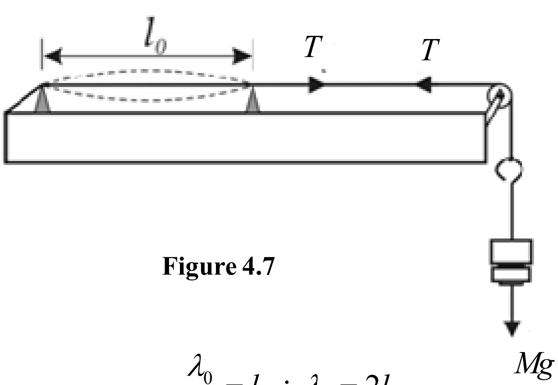


Figure 4.7

Figure 4.7 shows a similar situation. If the M is the mass suspended the tension in the string $T = Mg$ (neglecting the friction from the pulley)

If λ_0 is the wave length of the stationary wave formed along the string,

$$\frac{\lambda_0}{2} = l_0 \therefore \lambda_0 = 2l_0$$

$$f_0 = \frac{1}{2l_0}\sqrt{\frac{T}{m}}$$

The wire resonated since the frequency of the wire equalled the frequency of the tuning fork. The mass per unit of the string can be found by removing the wire, measuring its mass and length and then dividing the mass by length.
The frequency of the tuning fork can be found from,

$$f_0 = \frac{1}{2l_0}\sqrt{\frac{Mg}{m}}$$

Finding the relation between the resonance length and frequency of the string using the sonometer

The set of tunning fork with known frequency in the labrotary can be used for this. Keeping the tension of the string constant , vibrate the tuning fork with the maximum frequency (512 Hz) and place it on the sonometer box. Then increase the vibrating length of the string by sliding the knife edge, starting from the shoretest length position of the string so that the fundamental resonance situation (l_0) is obtained (use the paper rider to obtain the resonance situation).

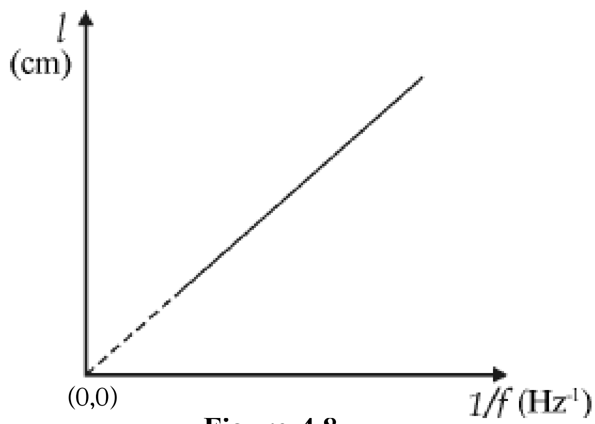


Figure 4.8

Then keeping the tenson T of the string constant, find the resonance lengths (l) corresponding to the frequencies of the other tunning forks by selecting them in the ascending order of frequencies. Plot l against $1/f$.

Then you will get a straight line passing through the origin. Therefore, when T is constant the length l is inversely propational to f.

Velocity of longitudinal waves a medium

When a wave propagates along a string the instantaneous deformations returns to normal due to the tension in the wire. But such a tension does exist in any medium and hence the deformations that occur when a wave travels in such a medium returns to normal due to the elastic property of the medium.

The velocity of longitudinal waves in any medium is given by,

$$v = \sqrt{\frac{E}{\rho}}$$

where E is the modulus of elasticity of the medium and ρ its density.

The modulus of elasticity for a solid medium is the Young's modulus. For a liquid medium the relevant modulus for the velocity of longitudinal waves is the bulk modulus.

If E is the Young modulus and ρ the density of a solid medium, the velocity of longitudinal waves in the medium is given by,

$$v = \sqrt{\frac{E}{\rho}}$$

Since the propagation of sound through solid media also takes place as longitudinal waves, the above equation is valid for the velocity of sound through solid media. For example, the Young's modulus for wrought iron is $197\times10^{9}\ \mathrm{N\ m^{-2}}$ and its density is $7850\ \mathrm{kg\ m^{-3}}$. Hence the velocity of sound through iron is,

$$v = \sqrt{\frac{197\times10^{9}}{7850}} = 5009.5\ \mathrm{m\ s^{-1}}$$

This value is in agreement with the practically determined value of the velocity of sound in iron which is $5000\ \mathrm{m\ s^{-1}}$.

The velocity of longitudinal waves in liquids too can be found by using this relationship. For example the bulk modulus of water is $2.05\times10^{9}\ \mathrm{N\ m^{-2}}$ and the density is $995.5\ \mathrm{kg\ m^{-3}}$ at $27\ ^{o}$C.

$$\therefore v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{2.05\times10^{9}}{995.5}} = 1433.2\ \mathrm{m\ s^{-1}}$$

$\therefore$ Experimental value for velocity of sound at $27\ ^{o}$C is $1498\ \mathrm{m\ s^{-1}}$.

Seismic waves

Seismic waves are produced by intense vibrations or explosions taking place inside Earth. On many occasions these vibrations are caused by earthquakes. Although we in Sri Lanka do not have the experience of a major earthquake, we have heard of vast damages and loss of life caused by earthquakes that occurred in various other locations on the Earth.

A large number of earthquakes take place on Earth every year. Since the intensities of these vary from each other, the destructions caused by them also vary from each other. The most intense earthquake recorded had occurred in Chilsa (9.5 on Richter scale) in the 22^{nd} of May 1960 with a death toll of 4000- 5000 . According to historical records the largest death toll (830,000) occurred in China on the 23^{rd} January 1556 in an earthquake of lesser intensity than that occurred in Chilea. Although we haven't encountered any direct earthquake and its consequences, the tsunami that occurred in the Indian Ocean on the 26^{th} of December 2004 caused immense devastation.

Since there is a tendency of occurring an earthquake near Sri Lanka and also a tsunami due to an earthquake in the Indian Ocean at any instant, it is important that we in Sri Lanka should have an awareness of this situation. Also it is possible to predict of weather gales etc. No technology has been able to predict yet about earthquakes. What makes all destructions in an earthquake are the seismic waves produced by the earthquake.

There are two main reasons responsible for earthquakes. The interactions between internal tectonic plates, is the main reason for earthquakes while the other reason is the vibrations due to volcanic eruptions. But the earthquakes due to volcanic eruptions are relatively less in extent than those due to interactions between tectonic plates. In addition, seismic waves also erupt due to explosions of atomic bombs on various parts of Earth. These explosions can be identified as small earthquakes, an example being the testing of a bomb of TNT 5 HL power by America on 6^{th} November 1971.This explosion caused an earthquake of reading 6.9 on the Richter scale.

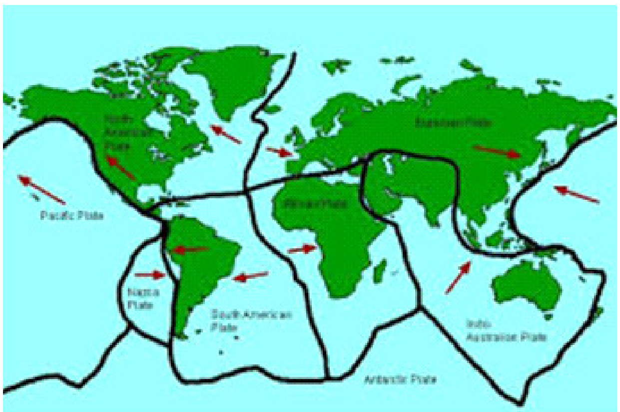
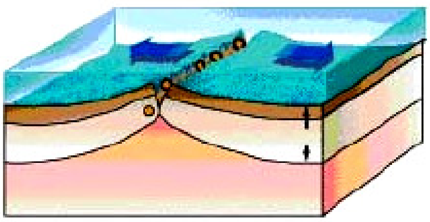


Figure 4.9

The tectonic plates of earth surface are thickly formed. These plates freely float very slowly on liquid magma in the core of the Earth. The Figure 4.9 shows the main tectonic plates on the Earth surface.

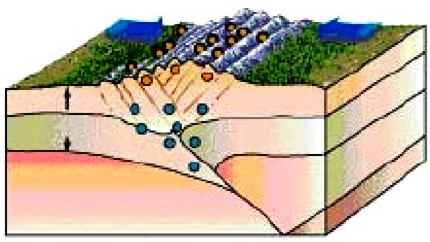
Due to the interactions between these plates thay undergo elastic deformations. At an instance when this deformation reaches an unbearable level, that portion of the Earth undergoes a sudden explosion. This explosion emits a vast amount of energy which has been acquired from a long period of time. The result is an earthquake.

The interactions between tectonic plates can take place in four different ways which are illustrated in Figure 4.10.



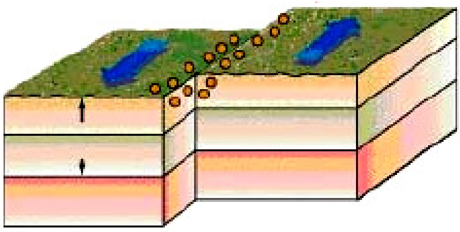
Divergent Boundary

Figure 4.10 (a)



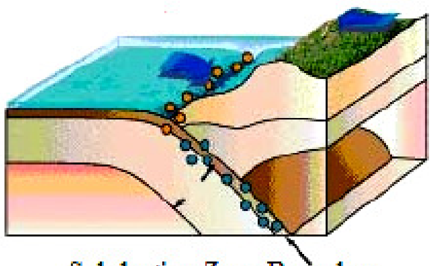
Continental Collision Boundary

Figure 4.10 (b)



Transform Fault Boundary

Figure 4.10 (c)



Subduction Zone Boundary

Figure 4.10 (d)

Figure 4.10 (a) shows the approach of tectonic plates towards each other causing their edges sliding upon each other to commence and earthquake. Here too sudden explosions occur in Earth due to unbearable deformations.

Figure 4.10(b) shows a sudden vertical displacement of one tectonic plate. If an earthquake of this type occur in the sea, in addition to the earthquake, a tsunami can occur causing a huge devastation.

Figure 4.10(c) shows the horizontal displacements between the tectonic plates. Here the Earth surface undergoes a horizoutal deformation and there is a sudden emission of a vast amount of energy causing a horizontal movement of the plates.

Figure 4.10 (d) shows a sudden vertical displacement of one tectonic plate. If an earthquake of this type occur in the sea, in addition to the earthquake, a tsunami can occur causing a huge devastation.shows how earthquakes are produced when tectonic plate move away from each other. During these earthquakes large clefts are formed on the ground.

When an earthquake occurs releasing deforming forces, the position of the explosion in the Earth is known as the 'hypocenter' or the 'focus'. This can be situated a number of kilometers under the Earth surface. The point on the Earth surface vertically above the hypocenter is known as the 'epicenter' when the depth of the hypocenter below the ground decreases the intensity of the earthquake increases.

The damage due to an earthquake spread due to eruption of seismic waves. The seismic waves produced in an earthquake can be subdivided into two types namely 'body waves' and 'surface waves'.

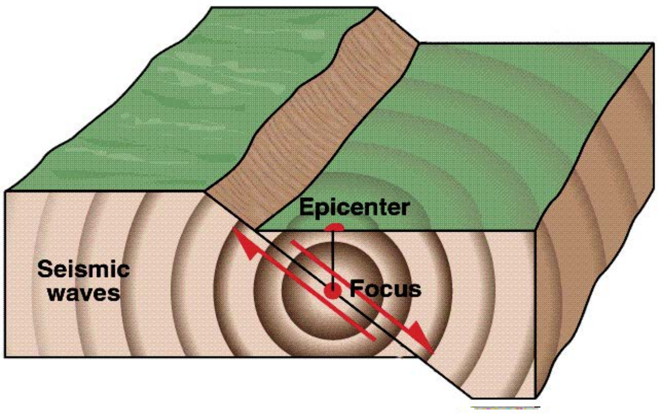


Figure 4.11

Body waves

Body waves have a higher speed than surface waves. Hence equipment measuring earthquakes first sense body waves. Also body waves have a frequency higher than that of surface waves. According to propagation of body waves they can further be subdivided into two more types.

P (Primary) waves

Of the body waves the first to arrive at the seismic station are the P waves and hence they are also called primary waves. These waves propagate longitudinally as sound waves. Hence P waves can travel through all three media, solid, liquid and gas. The velocity of P waves in all these media is equal to the velocity of sound in these media. Accordingly the velocity of P waves in solid (rocks) medium is 5000 m s^{-1}, in liquid (water) is and in air is about 330 m s^{-1}. Figure 4.12 illustrates the movements of the particles of the medium during the propagation of this wave. The P waves produced in an earthquake are heard by animals such as elephants and dogs, but not by humans. Humans sense only the vibration of an object.

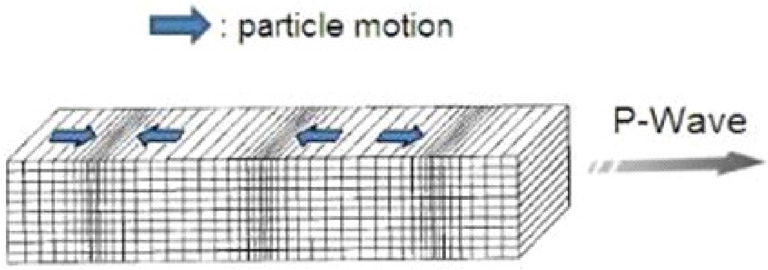


Figure 4.12

S waves

S waves are the second to arrive at a seismic station after an earthquake and are hence known as secondary waves. The velocity of these waves is about 60% of the velocity of P waves. The main difference between these waves and the P waves is that these waves are transverse. These transverse waves vibrate in horizontal as well as in vertical planes.

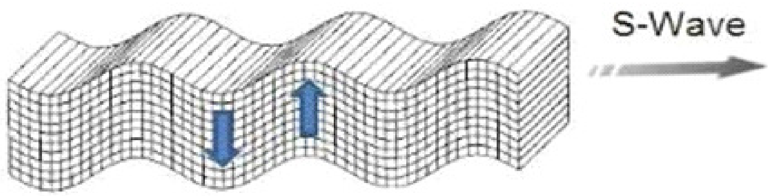


Figure 4.13

Since these are transverse waves they can travel only through solid (rock) media. Relative to the direction of propagation of the wave, vibrations occur sideways as well as up and down. The Figure 4.13 shows the movements of the particles during the propagation of the wave.

Surface waves

Surface waves, the second type of seismic waves have a frequency less than that of body waves. Also their velocity too is relatiivily less. The velocity of surface waves is about 90% of the velocity of S-waves. Surface waves are responsible for the main part of the destructions done by an earthquake. Surface waves are subdivided into two main categories.

Rayleigh waves

The mathematical expression and the explanation about Rayliegh waves was first introduced by Lord Rayleigh in 1885 and as an honor waves are named after him. A vibration which is a mixture of both transverse and longitudinal vibrations of the particles of the medium guide the propagation of the wave which travels at a speed of 50-300 m s^{-1} on the Earth surface.

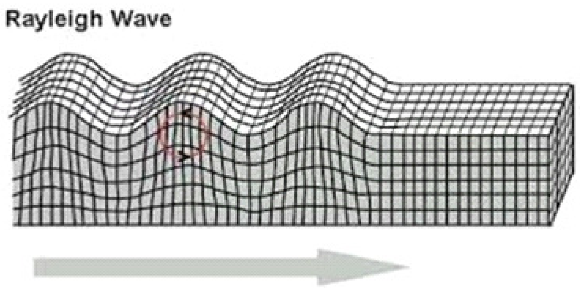


Figure 4.14 – Rayleigh wave

These waves originate as a result of the interactions between the P waves and S waves coming from the hypocentre which is the vibrating point inside the Earth.

Love waves

These waves belong to transverse type and execute only vibrations in the horizontal plane. The mathematical explanation about these waves was first done by A.Z. H. Love in 1911 and as an honor to this scientist the waves are named after him. The velocity of Love waves is slightly greater than that of Rayleigh waves. The figure indicates the movements of the particles of the medium in which the wave travels.

Richter scale

Since an enormous amount of energy spreads over a large range during an earthquake, it is difficult to use a linear scale to measure the intensity of the earthquake. The amplitude of the waves produced by an earthquake is used to measure the energy of the vibration and the logarithm of the maximum amplitude of the vibration produced is used in the Richter scale to measure the strength of the vibration of the earthquake. Thus an increase by a unit of the reading of a Richter scale indicates an increase by 10 times of the strength of an earthquake. For example, a reading recorded as 9 indicates an earthquake 10times stronger than that recorded as 8. Also a unit increase on the Richter scale means 30 times increase of the energy produced.

Tsunami

A series of waves of enormous energy produced as a result of an earthquake in the sea on reaching the land causes a Tsunami. These waves lead to vast destruction of property and loss of life. The word "tsunami" comes from a Japanese word of meaning "harbour wave". Some interpret this phenomenon incorrectly as "tidal waves". But tidal waves are due to the gravitational attractions on sea water surface by the moon and the sun. Normal sea waves are due to wind. There could be many reasons for the occurrence of a tsunami such as,

(i) Earthquakes inside the sea
(ii) Volcano eruptions in the sea
(iii) Slides in the sea bed
(iv) Asteroids crashing into water from space

If an earthquake is to cause a tsunami, certain conditions have to be fulfilled. The earthquake should occur inside the sea and should register a minimum reading of 6.75 on the Richter scale.

Occurrence of earthquakes by the collision of tectonic plates was explained earlier. As illustrated in Figure 4.15, a sudden upward movement of a tectonic plate causing an earthquake is another condition for the occurrence of a Tsunami. This is why that every earthquake in the sea does not produce a Tsunami.

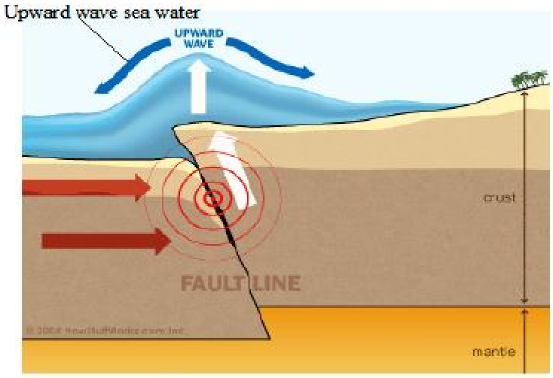


Figure 4.15

The Figure 4.15 shows how plates move causing the formation of a large mass of water on the water surface. The amplitude of this disturbance of water in the deep sea is less than 1 m most of the time. When this disturbance which occurs in the deep sea spreads the resulting transverse wave has a wavelength of about 100 km. Also the periodic time of the wave could be as long as an hour. Due to this reason it becomes very difficult to identify a tsunami in the deep sea. However when the tsunami wave reaches the land it is subjected to a few changes.

Those changes are as follows.

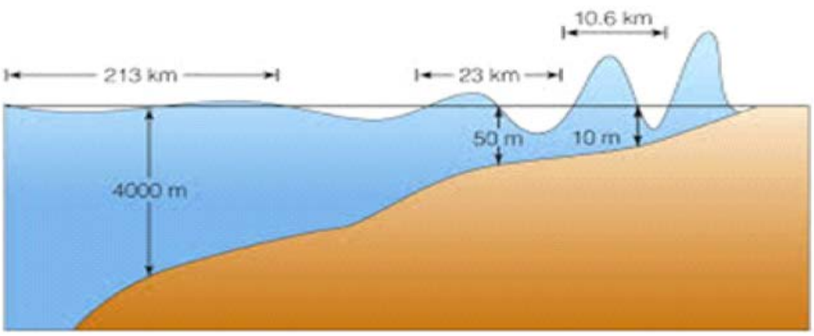


Figure 4.16

1. The velocity (v) of the tsunami wave decreases when reaching the sea shore.
2. The wavelength (λ) too of the wave decreases.
3. But the amplitude (a) of the wave increases.

The Figure 4.16 shows the changes occurring on the water surface when reaching the sea shore. Due to the enormous amount of energy possessed by the tsunami wave reaching the shore and the high amplitudes of the wave, it can cause immense destruction to the sea shore. This is why a tsunamic can lead to vast damages of property and loss of life.

A tsunamic that occurred in the Indian occur near Sumatra in 2004, raised the amptitude of the wave to about 30 m at certain places on the shore. These waves reached high amptitudes on the Western and Southern shores of Sri Lanka too.

According to records, the tsunami which reached the Southern coast of Sri Lanka on 26th December 2004 had a wave of height 10 m at Kahawa, 9 m at Koggala, 8.7 m at Nonagama and 6 m at Galle and Payagala.

It is possible to explain the changes (decrease of velocity and wave length and increase of amplitude) mentioned above, which occur when tsunami waves reach the shore, by using the properties of waves. But since this subjects matter is beyond the scope of the Advanced Level syllabus, it has been included at the end of this chapter as extra knowledge.

Before the arrival of tsunami waves to the sea shore there is an unusual lowering of the water level there. This happens due to the pulling of water to the regions of high displacements. Since the periodic time of a tsunami has a high value, the evacuation of water lasts for some time. Then the section of high displacement rushes towards sea shore to cause the maximum damage. It is important to be aware of these characteristics of a tsunami in order to escape from or minimize the damages cause by it.

Chapter - 05

Transmission of sound through gases

It has been confirmed that a material medium is essential for the transmission of mechanical waves. Since sound too is a kind of a mechanical wave, which in general transmits as longitudinal waves, it can be concluded that sound too needs a material medium for transmission.

The velocity of any mechanical wave through a medium depends on both the inertial properties and the elastic properties of the medium. Accordingly, the velocity of sound through a medium is expressed as,

$v = \sqrt{\frac{E}{\rho}}$,where ρ is the density of the medium and E is the relevant modulus of elasticity of the medium.

When considering how sound travels through a gaseous medium it can be shown to be a flow of compressions and rarefactions, which is in the form of a longitudinal wave. The modulus of elasticity relevant to these compressions and rarefactions is the bulk modulus.

However a gas has two bulk moduli.

One is $E = p$, where p is the pressure, for gradual changes of pressure and volume under isothermal conditions.

The other is $E = \gamma p$ where $\gamma = \frac{c_p}{c_v}$ is the ratio of principle specific heat capacities of the gas and p is the pressure, for rapid changes of pressure and volume under adiabatic conditions.

Sir Isaac Newton who played a prominent role in these investigations of determining the velocity of sound through gases, used $E = p$ (pressure of the gas) in the above equation but found that the value obtained by means of these calculations differed considerably from the value (330 m s^{-1}) obtained experimentally.

This problem remained unsolved for nearly about one hundred years, until a scientist named Laplace used the relation $E = \gamma p$, for the bulk modulus corresponding to adiabatic changes in the equation for velocity of sound. Using this approach he obtained a value for the velocity of sound, which agreed well with the practically obtained value.

Hence, it was confirmed that the velocity of sound in gases is expressed by the formula,

$$v = \sqrt{\frac{\gamma p}{\rho}} \quad \text{.................................5.1}$$

Taking density $\rho = \dfrac{mass\ (m)}{volume\ (V)}$

$$v = \sqrt{\frac{\gamma p}{m/V}}$$

$$v = \sqrt{\frac{\gamma pV}{m}} \quad \text{............................5.2}$$

where V is the volume of a mass m of a gas and p is the pressure.Then considering one mole of the gas,

Where, $pV = RT$ and $m = M$ (mass of one mole of the gas)

Then $v = \sqrt{\dfrac{\gamma RT}{M}}$ …………........5.3 is another expression for the velocity of sound in a gas.

Accordingly, the velocity of sound in a gas is,

(1) independent of pressure
(2) dependent on temperature with the velocity increasing with increasing temperature.

According to the equation 5.3, $v = k\sqrt{T}$

If v_1 and v_2 are the two velocities that correspond to temperatures T_1 and T_2 respectively.

$$v_1 = k\sqrt{T_1}$$

$$v_2 = k\sqrt{T_2}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \quad \text{..............................5.4}$$

Solved Example

A tuning fork of frequency 256 Hz emits a sound wave of wavelength 136 cm at a temperature of 30 ^{o}C. Find the velocity of sound in air at 30 ^{o}C. Hence, find the velocity of sound at S.T.P. If the density of air at S.T.P. is 1.293 kg m^{-3} and the standard pressure is

10^5 N m^{-2}, calculate the ratio of the principal specific heat capacities $\left(\gamma = \dfrac{c_P}{c_V}\right)$ of air.

Solution

$$v_{30} = f\lambda_{30} = 256 \times 1.36 = 348 \text{ m s}^{-1}$$

$$\frac{v_0}{v_{30}} = \sqrt{\frac{T_0}{T_{30}}} = \sqrt{\frac{273}{303}}$$

$$\therefore v_0 = v_{30}\sqrt{\frac{273}{303}} = 348 \times 0.949 = 330 \text{ m s}^{-1}$$

$$v_0 = \sqrt{\frac{\gamma P}{\rho}}$$

$$\therefore 330 = \sqrt{\frac{\gamma \times 10^5}{1.293}}$$

$$\gamma = \frac{330^2 \times 1.293}{10^5} = 1.40$$

Vibration of gas columns

Close resonant tube

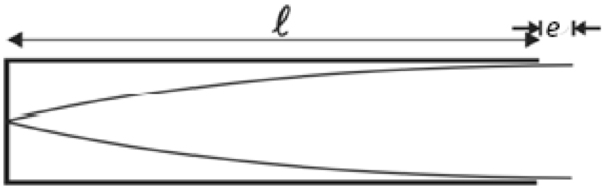
A column of air inside a tube closed at one end, can be made to vibrate by a source of sound placed at the open end of the tube to form a stationary wave in the tube. The sound wave which is emitted as a longitudinal wave after transmission through the air column gets reflected at the closed end. This reflected wave interfere with the incident wave to form a stationary wave. However, for this to happen the length of the tube should get adjusted to suit the wave length of the sound wave passing through the respective air columns in the tube. At this stage, the frequency of vibration of the air column in the tube equals the frequency of the sound wave from the source. Due to the space available for vibrations of the air column it vibrates with a large amplitude emitting a sound wave of same frequency as that emitted by the source of sound, but of higher intensity. This phenomenon has been used as the principle in pipe type musical instruments such as organs.

These stationary waves formed inside a resonating tube should essentially produce a node at the closed end of the tube, since air molecules have no space to vibrate in the direction of the wave at this end. However, due to the large space available at the open end of the tube for vibration of air columns an antinode is formed at this end, while getting pushed out a little at this point.

The simplest stationary resonant wave formed inside a tube closed at one end is known as the "fundamental" or the first harmonic. Additional modes of resonant waves can be formed in the tube by gradually increasing the frequency of the source while keeping the length of the tube constant.

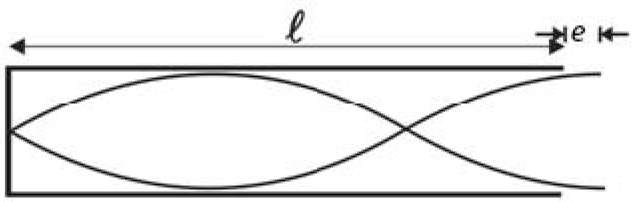
In the following representation of such wave modes, vibrations of the gas molecules occur in the forward and the backward directions parallel to the direction of the wave. These vibrations are in general indicated along the *y* axis. The small displacement of the antinode of the wave at the open end of the tube is known as the end correction (e) of the tube and gets added to the effective length of the tube.

(1) Fundamental / First harmonic



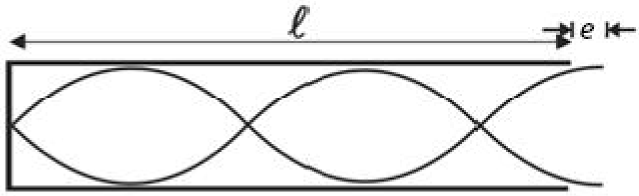
$$\frac{\lambda_1}{4} = l + e \Rightarrow \lambda_1 = 4(l+e)$$

Figure 5.1



$$\frac{3\lambda_2}{4} = l + e \Rightarrow \lambda_2 = \frac{4}{3}(l+e)$$

Figure 5.2



$$\frac{5\lambda_3}{4} = l + e \Rightarrow \lambda_3 = \frac{4}{5}(l+e)$$

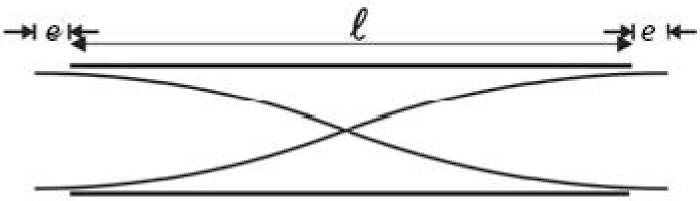
Figure 5.3

This clearly shows that, if the length of the tube could be varied, after obtaining the fundamental resonance, the other states of resonance shown above can be obtained by increasing the length of the tube approximately three times, five times, etc. of the original length keeping the frequency of the source constant.

Open resonant tube

Even in a tube open at both ends, a sound wave admitted from one end when reaching the other open end gets reflected by a certain amount. Hence here too if the length of the tube could be adjusted to suit the wave length of the wave, resonance states can be obtained as follows;

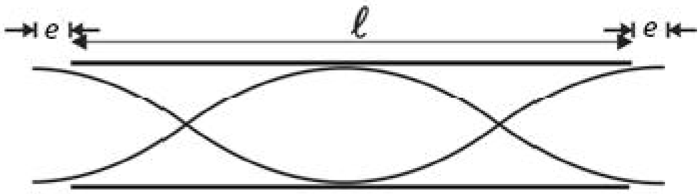
(1) Fundamental / First harmonic



$$\frac{\lambda_1}{2} = l + 2e \Rightarrow \lambda_1 = 2(l + 2e)$$

Figure 5.4

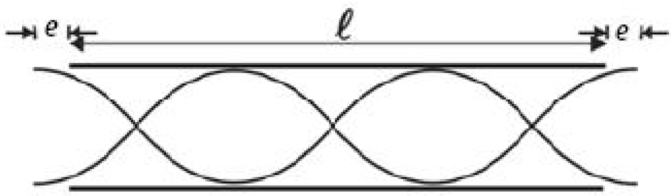
(2) First overtone / Second harmonic



$$\lambda_2 = l + 2e$$

Figure 5.5

(1) Second overtone / Third harmonic



$$\frac{3\lambda_3}{2} = l + 2e \Rightarrow \lambda_3 = \frac{2}{3}(l + 2e)$$

Figure 5.6

By making use of the above states of resonance in resonance tubes, experiment can be designed to determined the velocity of sound in air in the school laboratary.

Worked example

A tuning fork of frequency 320 Hz is made to vibrate and is held above the open end of a closed resonance tube of which the length can be varied. The first state of resonance is reached when the length of the air column is 25.3 cm. For a tuning fork of frequency 480 Hz, the length of the air column of the same tube, to produce the same state of resonance, is 16.5 cm. Find from these observations the velocity of sound in air and the end correction of the resonance tube.

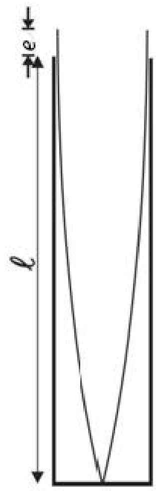
In the fundamental $\frac{\lambda}{4} = l + e$

$$\lambda = 4(l+e)$$

According to $v = f\lambda$, $\lambda = \frac{v}{f}$

$$\therefore 4(l+e) = \frac{v}{f}$$

$$l + e = \frac{v}{4f}$$



For the first turning fork $25.3 + e = \frac{v}{4 \times 320}$ ....(1)

For the second turning fork $16.5 + e = \frac{v}{4 \times 480}$ ....(2)

(1) - (2) gives $8.8 = \frac{v}{4}\left(\frac{1}{320} - \frac{1}{480}\right)$

$$v = 337.9 \text{ m s}^{-1}$$

Substituting for v in equation 5.5 or 5.6

$$e = 1.1 \text{ cm}$$

Chapter - 06

Doppler effect

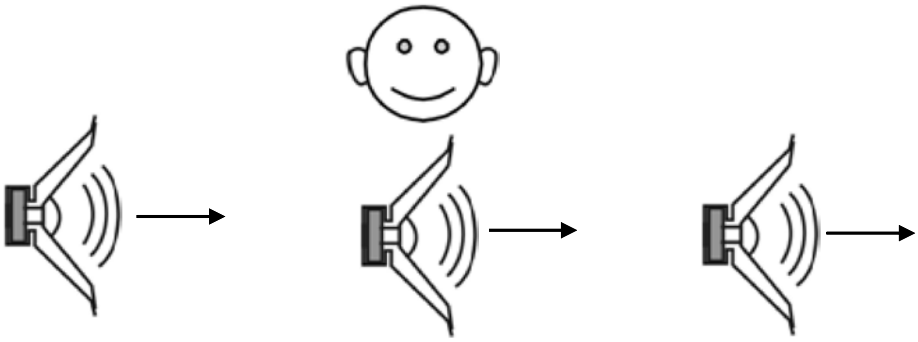


Figure 6.1

Imagine that when you are standing by the side of a railway track a train is approaching you along the track sounding its horn with constant frequency f_0. If the train is moving with a uniform velocity, what will be your observation regarding the sound you hear from the horn?

As the train is approaching you the intensity level (loudness) of the sound emitted by the horn will gradually increase, reach a maximum when the horn of the train passes you and then gradually decrease as the train moves away from you.

When the train approaches you, the pitch of the sound you hear from the horn will not be that relevant to the natural frequency f_0 of the horn, but a pitch corresponding to a frequency higher than that of f_0.

If the train is approaching you with a uniform velocity, this higher pitch of a higher frequency will remain constant as the train approaches you but will suddenly drop to a pitch corresponding to a lower frequency than that of f_0, when the horn of the train passes you.

After the horn passes you, the lower pitch will remain constant as the train moves away from you.

The second phenomenon described above is known as "Doppler effect". It is not only in a situation as described above that Doppler effect is observed, but also when there is any type of relative motion between the source of sound and the observer.

Hence Doppler effect is the apparent change of frequency (pitch) of the sound from a source as heard by an observer when there is any relative motion between the observer and the source emitting the sound.

Doppler effect was revealed by Austrian scientist Johann Doppler in 1845.

Main situations of occurrence of Doppler effect

1. **Source moving towards an observer.**

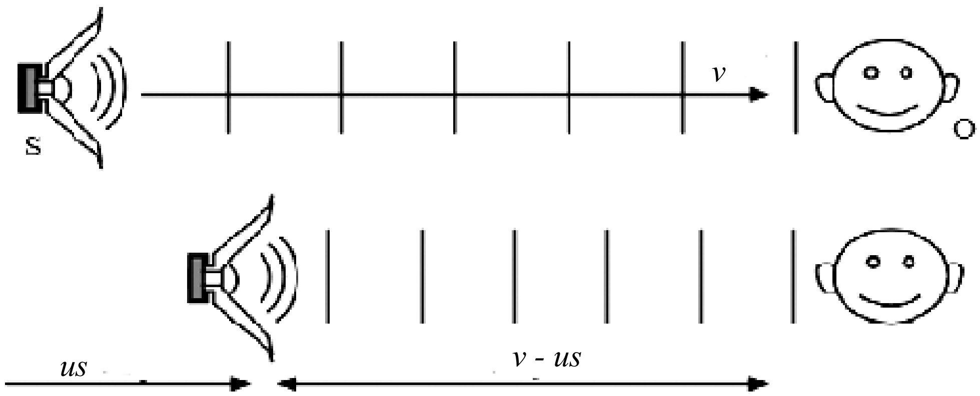


Figure 6.2

Suppose a source S is emitting sound of frequency, f_0, towards an observer, O which is at rest. If v is the velocity of sound in air then the wave length of the sound reaching the observer O,

$$\lambda_0 = \frac{v}{f_0}$$

Now, if the source is moving towards the observer with a velocity u_s, then f_0 number of waves get packed within a distance v-u_s, thus changing the wave length of the wave to a value.

$$\lambda = \frac{v - u_s}{f_0}$$

This would result in the change of the frequency of sound heard by the observer to an apparent value

$$f = \frac{v}{\lambda} = \frac{v}{\frac{v - u_s}{f_0}}$$

$$\therefore f = \left(\frac{v}{v - u_s}\right) f_0$$

It is clear that on this occasion the apparent frequency of the sound heard by the observer is greater than the natural frequency of the sound emitted by the source.

2. Source moving away from the observer.

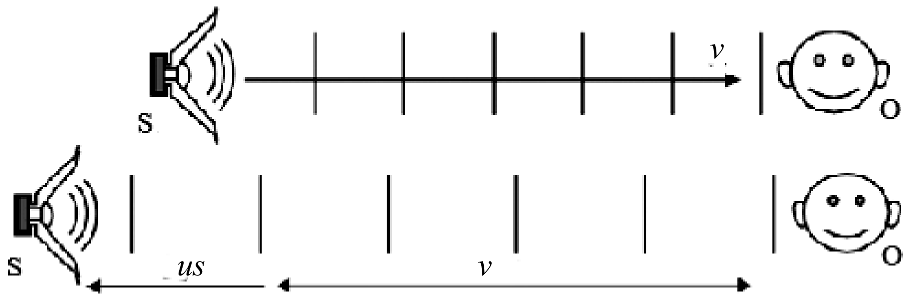


Figure 6.3

Suppose the source is moving away from the observer with a velocity u_s. Then an f_0 number of waves get stretched over a distance $(v + u_s)$, as a result of which the wave length of the sound wave reaching the observer changes to an apparent value,

$$\lambda = \frac{v + u_s}{f_0}$$

This would lead to a change in the frequency of the sound heard by the observer to an apparent value,

$$f = \frac{v}{\lambda} = \frac{v}{{v + u_s}/{f_0}} = \left(\frac{v}{v + u_s}\right) f_0$$

In this case the apparent frequency of the sound heard by the observer is less than the natural frequency.

3. Observer moving towards a stationary source

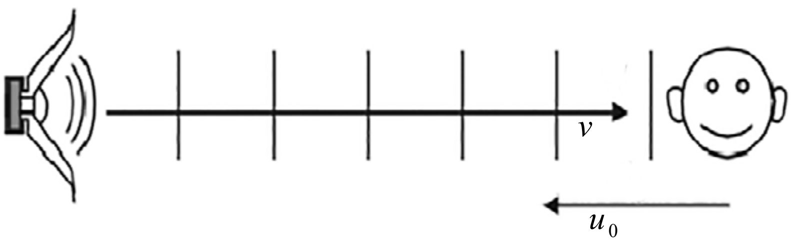


Figure -6.4

Suppose the source emitting sound of frequency f_0 is stationary while the observer is approaching the source with a velocity u_0. On this occasion, as the source is stationary the wave length of the sound wave emitted by it remains unchanged at $\lambda = \frac{v}{f_0}$. But since the observer is approaching the source with a velocity u_0, the velocity of sound relative to the observer changes to a value

($v+u_0$). This would result in the change of the frequency of the sound received by the observer to an apparent value,

$$f = \frac{v+u_0}{\lambda} = \frac{v+u_0}{v/f_0}$$

$$\therefore f = \left(\frac{v+u_0}{v}\right) f_0$$

Hence it appears that the observer hears a sound of higher frequency than of the natural frequency of the sound from the source.

4. Observer moving away from the source

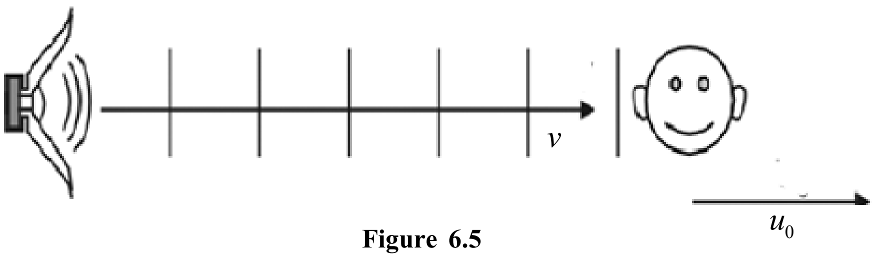


Figure 6.5

Suppose the observer is moving away from the stationary source with a velocity u_0 while the source emits sound of frequency f_0 and velocity v. This would result in the velocity of sound relative to the observer change to an apparent value $(v-u_0)$

This would cause the frequency of the sound received by the observer to change to an apparent value.

$$f = \frac{v-u_0}{\lambda} = \frac{v-u_0}{v/f_0}$$

$$\therefore f = \left(\frac{v-u_0}{v}\right) f_0$$

On this occasion, the frequency of the sound heard by the observer is less than the natural frequency of the sound from the source.

5. When both the source and the observer are moving relative to each other

On this occasion, the wave length changes due to the motion of the source and the velocity of sound relative to the observer changes due to his motion. If λ' is the apparent wavelength and v' the apparent velocity of sound relative to the observer then,

$$\lambda' = \frac{v \pm u_0}{f_0} \text{ and } v' = v \pm u_0$$

Then the apparent frequency of the sound heard by the observer,

$$f = \frac{v'}{\lambda'} = \left(\frac{v \pm u_s}{v \pm u_0}\right) f_0$$, where f_0 is the natural frequency of the source.

Doppler effect in light

In addition to sound, Doppler effect is observed in light too. Doppler effect is used to find out whether celestial bodies such as stars and galaxies are leaving us or approaching us. For this purpose the wavelength λ_0 of the light from a stationary body is determined first. Next, the wavelength λ of the light from a star or any other celestial body is found. If $\lambda > \lambda_0$ the wave length of light from the star has increased and the star is moving away from us. This is indicated by a shift of the spectral line corresponding to the wavelength λ towards the red spectral region of the spectrum. This shift of the spectral line towards the red line is referred to as the "Red shift".

On the other hand if $\lambda < \lambda_0$ the wavelength has decreased which means that the star is approaching us. The spectral line of λ has shifted towards the blue spectral region. This is referred to as a "Blue shift".

Applications of Doppler effect

1. Determination of translational and rotational motions of astronomical objects such as the sun and stars.
2. Measurement of speeds of vehicles using police radar.
3. Determination of speeds of blood cells.
4. Determination of speeds of aircrafts in airports.
5. Examination of the pulse of a fetus.

Worked example

A train engine B moves with a uniform speed of 3 m s^{-1} away from another engine A which is at rest. Both engines sound their whistles at 1000 Hz each.

(1) What is the apparent frequency of the sound from the whistle of B as heard by the driver in the stationary engine A?

(2) What is the apparent frequency of the sound from the whistle of A as heard by the driver of B?

(3) What are the beat frequencies heard by both drivers due to hearing of the sounds from their own whistles and of other engines?

(4) Now the train engine A moves in the same direction with a velocity of 1 m s^{-1}. Then what is the apparent frequency of the sound from the whistle of train B as heard by the driver of A? What is the beat frequency as heard by driver of A due to hearing of the sound of his own whistle and the sound from the whistle of B simultaneously?

(velocity of sound in air = 340 m s^{-1})

Solution

(1) u_B= 3 m s^{-1}, velocity of sound v = 340 m s^{-1}. Apparent frequency of sound B as heard by the driver of stationary engine A,

$$f_1 = \left(\frac{v}{v+u_B}\right) f_B = \left(\frac{340}{340+3}\right) 1000 = 991.3 \text{ Hz}$$

(2) Apparent frequency of sound from the stationary A as heard by B leaving A,

$$f_2 = \left(\frac{v-u_B}{v}\right) f_A = \left(\frac{340-3}{340}\right) 1000 = 991.2 \text{ Hz}$$

(3) Beat frequency as heard by the driver of A,

$= f_A - f_1 = 1000 - 991.3 = 8.7$ Hz

Beat frequency as heard by the driver of B

$= f_B - f_2 = 1000 - 991.2 = 8.8$ Hz

(4) u_A=1 m s^{-1}, u_B=3 m s^{-1}

Apparent frequency as heard by the driver of engine A following the engine B emitting sound of frequency 1000 Hz,

$$f_3 = \left(\frac{v+u_A}{v+u_B}\right) 1000 = \left(\frac{340+1}{340+3}\right) 1000 = 994.2 \text{ Hz}$$

Beat frequency as heard by A, $f = f_A - f_B = 1000 - 994.2 = 5.8$ Hz spread over a three-dimensional space.

Supersonic speeds; shock waves

Consider a source emitting sound of uniform frequency f_0 in an environment where the velocity of sound is v. The spherical wave fronts emitted by it spread over a three-dimensional space.

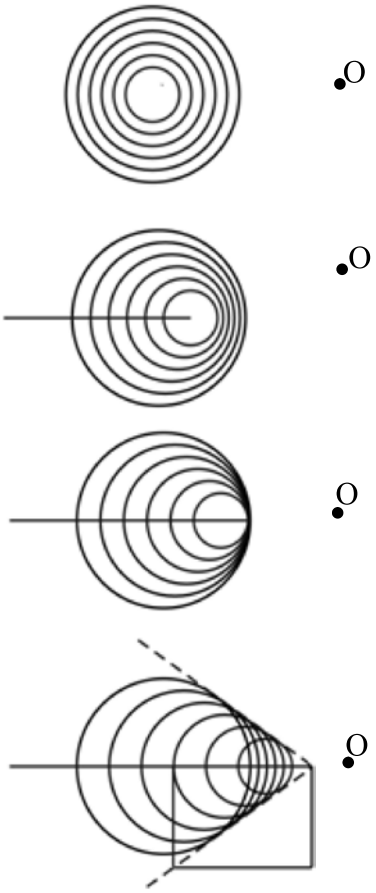


Figure 6.6

To a stationary observer standing close-by, the sound is heard with the same unchanged frequency f_0.

If the source now moves towards the observer with a velocity $u(<v)$ the apparent frequency of the sound heard by the observer, according to Doppler effect is $f = \left(\dfrac{v}{v-u}\right) f_0$ Hence $f > f_0$.

Now, if the speed of the source is further increased until it equals the velocity of sound ($u=v$), then, according to the above expression, the frequency of sound heard by the observer tends to infinity $f \rightarrow \propto$.

If the source now approaches the observer with a velocity exceeding the velocity of sound ($u>v$) then the above expression is no more valid. The wave-fronts spread as a three dimensional bunch packed inside a conical shaped envelope. This cone is known as the "Mach cone". Due to the abrupt rise and fall of air pressure because of this bunching effect of wave fronts a shock wave exists along the surface of this cone. This shock wave produces a burst of sound which is called a "Sonic boom", in which the air pressure suddenly increases and then decreases. Such high speeds are known as supersonic speeds.

Figure 6.7

An example where a shock wave is produced can be explained using the flight of a jet plane whose speed exceeds the velocity of sound. The shock wave emitted by it produces a sonic boom (Figure 6.7).

Shooting of a bullet from a rifle and snapping of a whip quickly making its tail move faster than the speed of sound are other instances where sonic booms are produced.

Chapter - 07

Nature of sound

Characteristics of sound

When we hear a certain sound, most of the time we are able to identify it, even if the source is out of our sight. That is if it is a human voice whether it is a female voice or a male voice and if not either, whether the sound is from any other creature. Furthermore, we are able to distinguish between the noise from a moving vehicle and a high pitched sound from a musical orchestra and all these without seeing the source.

It is with the use of the following "characteristics of sound" that we are able to make the above identifications.

1. Loudness
2. Pitch
3. Quality of sound or timbre

1. Loudness

Loudness at a certain location means the level at which the intensity of sound exists at that location. The intensity of sound at a given location is the rate at which sound energy passes normally across a unit area at that location (W m^{-2}). It depends on the amplitude (A) of the sound wave received by the observer and also on the distance (d) between the source of sound and the observer as given below.

$$\text{Intensity } (I) \propto A^2$$

$$I \propto \frac{1}{d^2}$$

The human ear can sence a very wide range sound intensities. It range from a very small intensity value of 10^{-12} Wm^{-2} to a very high value 1Wm^{-2}.

The minimum intensity that can be sensed by the human ear $\left(10^{-12}\ \text{Wm}^{-2}\right)$ is called threshold of hearing and the intensity which brings pain to the ear is called threshold of pain.

It has been experimentally proved that for this huge range the response of human ear variates logrithmically with the intensity.

Therefore, a quality called intensity level has been introduced to represent the above mentioned range. It is defined as follows. It's unit is decibel (dB).

$$\text{Intensity level of sound} \quad (\beta) = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where I - given sound intensity

I_0- threshold of hearing

According to the above definition ,

The intensity leval corresponding to threshold of hearing $\beta = 10\log_{10}\left(\frac{10^{-12}}{10^{-12}}\right)$

$$\beta = 0$$

The intensity leval corresponding to threshold of pain $\beta = 10\log_{10}\left(\frac{1}{10^{-12}}\right)$

$$= 120\,\text{dB}$$

That is the range of intensity level corresponding to human ear is $0 \rightarrow 120\ \text{dB}$.

2. Pitch

Pitch of a sound means how a particular sound note is sensed by the human ear and depends on the frequency of the sound wave. When the frequency of a sound wave increases its pitch too increases accordingly.

For example, if the octave of music is considered. The difference between the middle C_1 note and the high C_2 note is that the pitch of C_2 is being higher than that of C_1.

Another example of low pitch and high pitch is the difference between a male voice and a female voice, the pitch of the voice of the latter being generally higher than that of the former.

3. Quality of sound or timbre

Quality of a sound means whether it is just one pure sound note or whether it is a mixture of one note with its overtones/harmonics. Most sounds we hear, including our own voices, are not pure sound notes but mixtures of one note with its overtones (harmonics). If, for example, the same sound note played by a tuning fork, a violin or a piano are fed into a cathode ray oscilloscope, these are the graphical modes of the waves that we obtain.

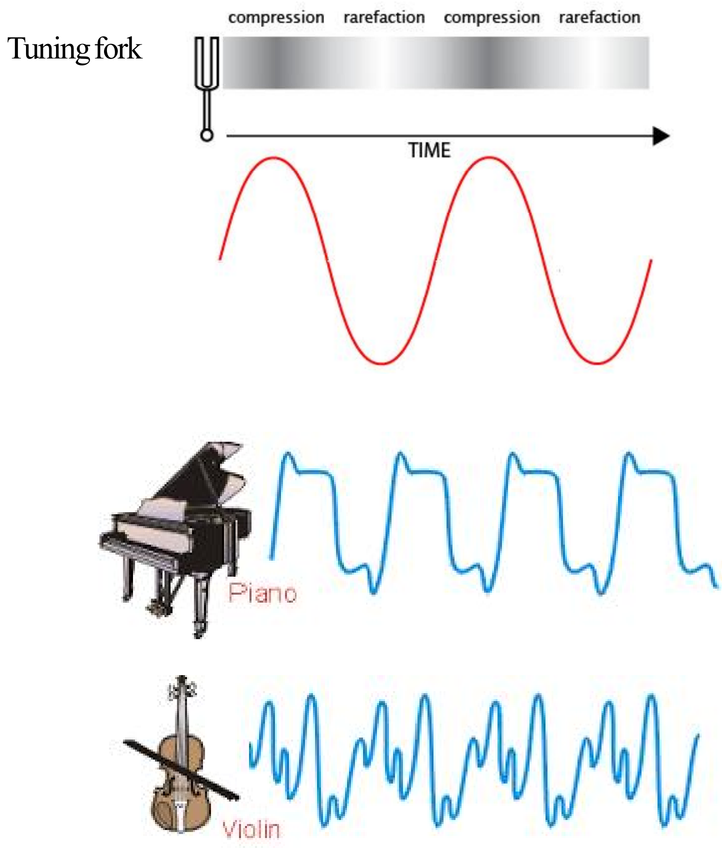


Figure 7.1

Although the same note was played by these instruments differences exist between their wave forms. Due to these differences in wave forms, they provide different sensations to the ear and these differences are attributed to the differences in the qualities of the sound.

The main difference between two female voices or between two male voices is also due to the differences between the qualities of sound of their voices.

Worked example

The sound intensity level at a certain point situated at a certain distance from a place where an electric drill is operating is 80 dB. If four similar electric drills are operating together at the same place, what would be the sound intensity level at the same point where the initial intensity level was 80 dB?

Solution

If I is the sound intensity at the given point due to one operating electric drill, then the sound intensity at the same point due to four operating drills = $4I$.

$\therefore$ Increase in intensity level $\qquad \beta = 10\log_{10}\left(\frac{4I}{I}\right)$

$$= 10\log_{10}(4)$$

$$= 10 \times 0.602 = 6.02 \text{ dB}$$

$\therefore$ New intensity level at the point $= 80+6.02$

$$= 86.02 \text{ dB}$$

Hearing Limits

The most important factors regarding hearing is how the human ear responds to loudness (intensity level) and pitch (frequency) of the sound that is heard. The human ear has ranges of response for both these quantities. For example, the range of frequencies audible to the normal human ear is considered to be from 20 Hz to 20000 Hz. The human ear does not respond to frequencies beyond the limits of this range.

It is also evident from the following graph that the levels of intensity (dB) too has to be adjusted

It is also evident from the following graph that the levels of intensity (dB) too has to be adjusted accordingly for proper hearing of sound within specific frequency ranges. According to this graph, for proper hearing of sound between 1000 Hz and 4000 Hz, very low intensity levels will be sufficient. Thus an intensity level as low as 20 dB is sufficient to hear a 1000 Hz sound conveniently. Sound of frequency 100 Hz can never be heard at this intensity level but needs a level of about 35 dB. However to hear sound of frequency as high as 20000 Hz the intensity level has to be as high as 40 dB

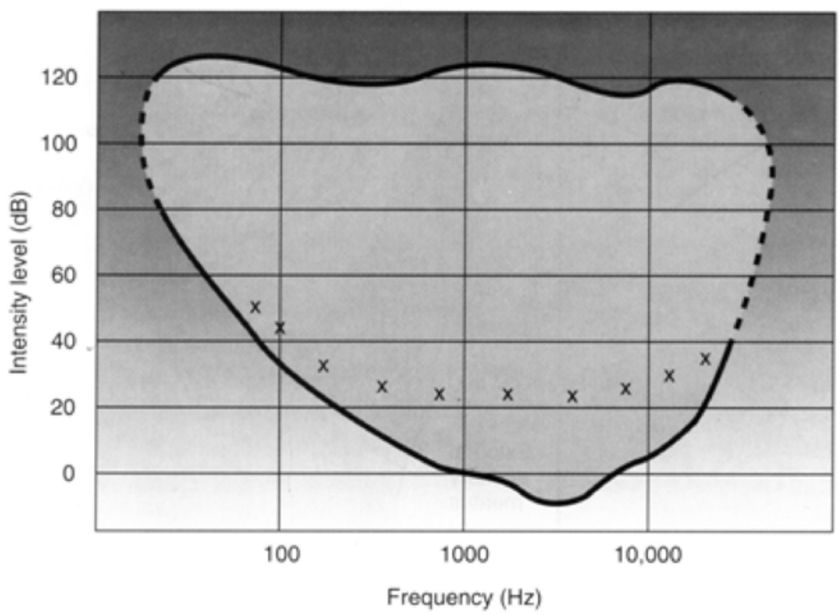


Figure 7.2-Graph of intensity level of sound versus the frequency of the sound for a human ear.

From the Figure 7.2, it can be seen that the region between 100 dB and 120 dB belongs to the threshold of pain while the region from 0 dB to about 20 dB belongs to the threshold of hearing.

The region of hearing frequencies decreases with the ageing of a person. Those who suffer from this deficiency use hearing aids to overcome it. For this purpose the person suffering from this defect is subjected to a special examination in which those frequency ranges where the person has poor hearing are detected. A hearing aid is then designed to amplify these frequencies. By this the sound of those frequencies are brought to the normal intensity levels of the person.

Usage of sound for communication is not confined to humans, but also to some animals and birds. Animals such as dogs are said to communicate at frequencies about 20000 Hz which is beyond the frequency range of humans. These sounds are called ultrasonics for various technological purposes.

1. Doctors with the help of Doppler effect use ultrasonic to determine the speed of flow of blood.
2. As it is not as harmful as X-rays, ultrasound is used to obtain the details of a fetus.
3. Another usage in the field of medicine is to remove brain tumours with least harm to surrounding tissues.
4. Chiropractors and physical therapists use ultrasound to relieve lower back pain.

Chapter - 08

Electromagnetic waves

We already know that mechanical waves propagate in a medium with the help of the vibrations of the particles in the medium. On the other hand, electro-magnetic waves propagate due to an electric field and a magnetic field vibrating in two planes perpendicular to each other. Figure 8.1 represents such an electro-magnetic wave, of which electric field is denoted by E, while the magnetic field is denoted by B. The vibration of E takes place in the plane xy, while the vibraton of B takes place in the plane xz. The progression of the wave takes place along the x direction, perpendicular to both vibrating fields.

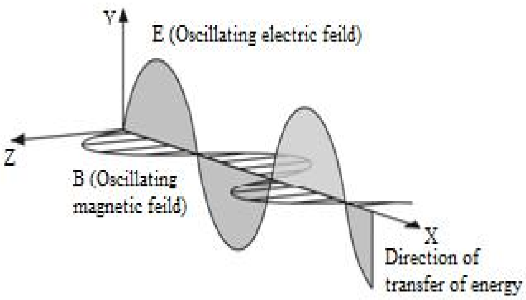


Figure 8.1

Both fields, E and B, exist in the same phase.
(By using more advanced theory it can be proved the velocity (c) of electro-magnetic waves is equal to the ratio between the amplitudes of the two fields.

$$c = \frac{E_0}{B_0} \qquad \text{and also given by} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

where μ_0 and ε_0 are the permeability and permittivity of free space.)

No medium is necessary for the progression of electromagnetic waves, since the fields E and B do not need any medium for their transmission. All electromagnetic (E.M) waves are found to travel with a velocity of 299,792,758 m s^{-1}, in a vacuum. For convenience, this experimentally found value is normally used as 3 x10^8 m s^{-1} in calculations. Electromagnetic waves can travel through several other media as well. But in these media the velocity of E.M. waves is less than that in a vacuum.

It has been shown that electro-magnetic (e.g. light) waves undergo plane polarization. It has been found that electric fields are mainly responsible for many processes, such as exposure in photography, fluorescence, etc. Hence the plane of a polarized electric field is taken as the standard plane of polarization of an electro-magnetic wave. Since, electromagnetic waves undergo polarization these waves are seen to behave as transverse waves.

According to the manner in which E.M. waves are produced their planes of vibration can exists in different orientations. For example light from a filament bulb consists of vibrations occurring in all the planes. In fact, it has been shown practically that the plane of polarization of light changes once in every 10^{-9} s.

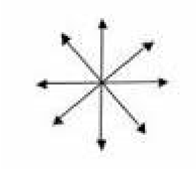


Figure 8.2(a)

The Figure 8.2(a), shows vibrations in all the planes, but only two vibrations normal to each other are shown in most cases in the form of diagrammatic representations.

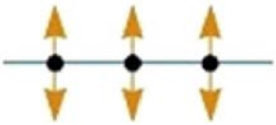


Figure 8.2(b)

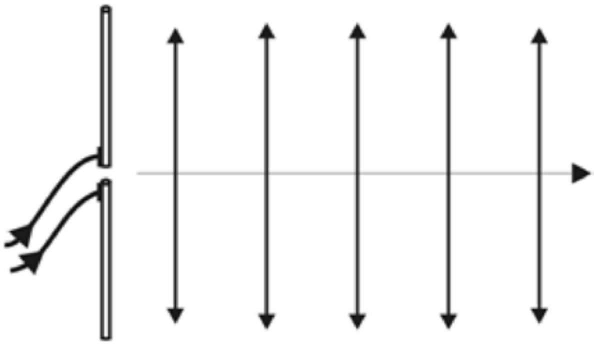


Figure 8.2(c)

As shown in Figure 8.2(c) the plane of polarization of the wave can be limited by allowing the wave to pass through a plane polarizer or by using a bi-polar antenna.

When electromagnetic waves pass through a medium, the particles of the medium absorb the wave and then emit it facilitating the motion of the wave. The time taken for this process differs from medium to medium. As such, electromagnetic waves travel with different velocities in different media.

There are a number of occasions in which electromagnetic waves are formed naturally, sun's radiation (UV, visible light, IR, etc,), are all electromagnetic waves reaching us naturally. The electric discharge that occurs during lightening is another source that produces E.M. waves. In the artificial production of E.M. waves, such as radio waves, electronic oscillating circuits take prominence of place. The other producers of E.M. waves are 'X-ray tubes', 'Magnetrons'. E.M. waves such as UV rays are also emitted during arc welding and carbon arcs. Mercury discharge lamps and fluorescent lamps also emit UV rays when those are illuminated. In all the above mentioned processes, E.M. waves are produced artificially. In addition, γ rays are emitted during nuclear reactions, during explosions of nuclear bombs and in nuclear power plants.

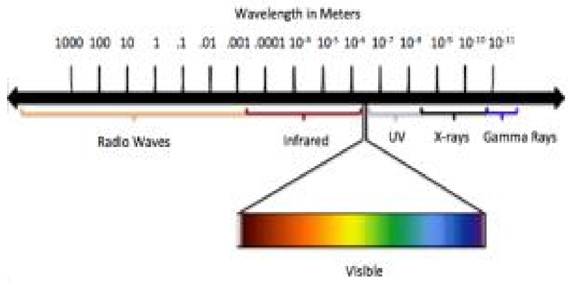
Electromagnetic spectrum

Electromagnetic waves spread over a wide range of frequencies (or wavelengths). This entire range can be divided into various frequency zones, which show properties different from each other. However, in a vacuum or in free space they progress with the same speed as the speed of light, which is given as 3×10^8 m s^{-1}. This entire range of frequencies of the electromagnetic waves divided into various zones is referred to as the electromagnetic spectrum.

The E.M. spectrum shown below indicates how the zones separate from each other according to frequencies (f) and wavelengths (λ). The E.M. spectrum is divided into six main zones. They are

1. Radio waves
2. Microwaves
3. Infra red radiation
4. Visible light
5. Ultra violet radiation
6. X – rays
7. γ - rays

(4–6) } Light



1. Radio waves

The zone extending from f=3 Hz ($\lambda = 10^8$ m) up to f= 3 THz ($\lambda = 10^{-4}$ m), belongs to the radio wave zone. The properties of these differ from each other while waves from different zones are utilized for different purposes. These zones are divided into several segments such as ELF, SLF, ULF, VLF, LF, MF, HF, VHF, UHF, SHF, EHF and THF according to the respective frequencies. The E.M. spectrum illustrated here, indicate these divisions clearly.

This radio wave zone is also sub divided as audio frequency zone and micro wave zone, with respect to the usage. Radio waves, in the audio frequency range are not audible to the human ear, unless the transducer (the speaker) convert these waves to sound waves.

2. Infrared radiation

The zone having frequencies immediately above radio wave zone is the infra-red zone. Those in the wavelength range from 10^{-2} m - 10^{-6} m belong to the infrared zone. Since the heat we receive from the sun is transmitted mainly as infrared rays, it is also referred to as heat radiation.

3. Visible light

The electromagnetic wave range sensitive to the eye is known as visible light. The wavelengths from 380 nm (violet) to 760 nm (red) belong to this range colour series red, orange, yellow, green, blue, indigo and violet which can be identified by the eye is called the visible spectrum. Infrared, visible light and ultra violet radiations are all classified as light.

4. Ultra violet radiation (UV)

The part having the frequencies above the purple is called ultraviolet radiation. These electromagnetic waves have the wavelength between 10^{-3} m - 10^{-9} m.

5. X-rays

The zone from $\lambda = 10^{-8}\ \text{m}\left(f = 3\times10^{16}\ \text{Hz}\right)$ to $\lambda = 10^{-13}\ \text{m}\left(f = 3\times10^{21}\ \text{Hz}\right)$ is known as X-rays. These rays can ionize media and are also capable of penetrating many media. X rays are produced by providing a high velocity to the electrons in the outer shells of atoms and then subjecting them to instantaneous decelerations.

6. γ-rays

The electromagnetic waves having frequencies above 10^{19} Hz belong to this category. They have high penetrating power. It can be seen that X-ray and γ ray parts overlap in the spectrum. The radiation emitted by the nucleus of the atom belongs to the γ-ray part and the rays generated by decelerating electrons belong to the X-ray part. The γ-rays ionizes the medium by a little amount.

The practical uses of electromagnetic waves in different ranges are briefly explained with the spectrum. External electric fields or magnetic fields do not affect the electromagnetic waves. Also these waves produce interference patterns. The patterns produces by X-rays are used to determine the way the atoms are arranged in a lattice. X-rays and γ-rays have high penetrating power.

LASERS

When the history of the Laser is written it will probably begin with that day in July 1960, when T.H.Maiman of the Hughes Aircraft Company in California carried out an experiment that was remarkably simple in concept, where a deep red beam of light, was generated from a synthetic Ruby rod. The term LASER is an acronym for Light Amplification by Stimulated Emission of Radiation. Its roots however, goes deeper to the principle of 'Stimulated Emission' presented by Albert Einstein in the year 1917. Due recognition should also be given to its Pioneers Charles Townes and Arthur Schawlow of the Colombia University, Weber of the Maryland University and the two Russian Scientists N. Basov and A.M. Prokhorov of the Lebedev Institute in Moscow.

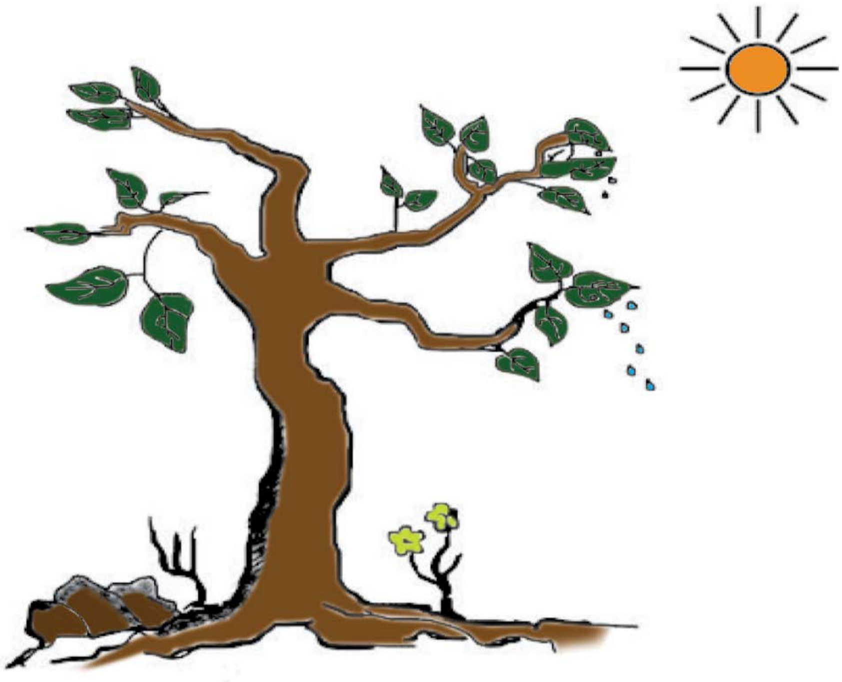


Figure 8.4

During the 2nd world war, Prof. Charles Townes on an invitation to visit the London war office went to a nearby park to spend some spare time early in the morning. As he looked up from his seat he observed that dew had formed on leaves of an Aeseala Tree in front of him. The sunlight was shining on the leaves and its heat made the dew droplets to combine with each other to form larger droplets that led to the formation of water drops. These water drops started to fall from leaves at the top of the tree onto those below them eventually forming a stream of water (Figure 8.4).This observation gave him the idea of using stimulated emission to generate a coherent beam of photons (or a beam of photons in step with each other) from a medium.

The Laser thus born was an unusual source of light, which was in many ways incomparably brighter than the sun. The beam consists of photons having the same energy and wavelength (monochromatic) and traveling in phase in the same direction (coherent).

Generation of a LASER beam

In the generation of a Laser beam from a material medium it is necessary to examine the processes that take place in the interaction of light with matter.

When a light beam interacts with a material medium three processes can take place. They are:

1.) Absorption 2.) Spontaneous Emission 3.) Stimulated Emission.

Of these, the process of Spontaneous Emission does not require the presence of any external energy source, such as a beam of light, for it to occur.

1) Absorption

In general, a large number of atoms (or molecules) in a stable material medium tend to occupy the ground energy states or its nearest neighbouring states. This situation remains unchanged if no external energy is incident upon it.

If an external light beam is incident on a material medium at equilibrium, then the atoms in their ground state absorb energy from the external beam and are excited to higher energy levels.

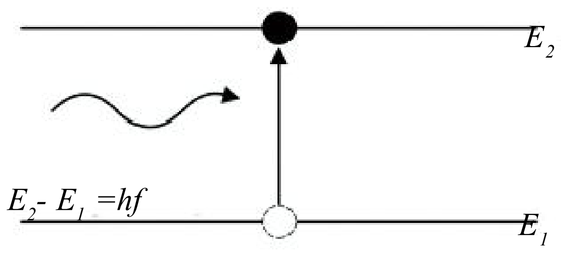


Figure 8.5

This energy, ΔE, absorbed by each individual atom will be in the form of a light photon, and the process of absorption will only take place if each photon possesses an energy, ΔE, exactly equal to the energy separation of the two levels (Figure. 8.5), given by

$\Delta E = E_2 - E_1 = hf$

where h is the Planck constant and f the frequency of the absorbed photon.

2) Spontaneous Emission

Now let us consider two energy levels of a material medium, one of which is a ground state of energy E_1 and the other an excited state of energy E_2.

Suppose by some means (that is by absorbing energy from an external source), an atom is excited from the lower energy level of energy E_1 to the upper energy level of energy E_2,

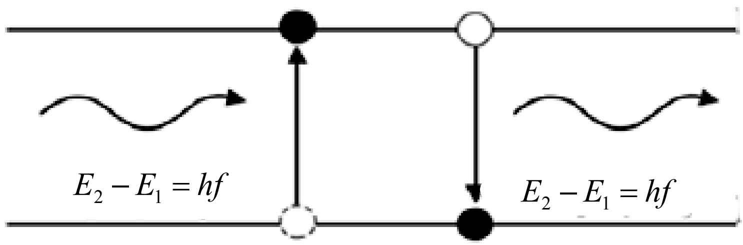


Figure 8.6

then its tendency would be to emit this extra energy it possess and fall back to its *ground state of energy* $E1$ (Figure 8.6). In this process of falling back (or reverting) to its ground state spontaneously, the excited atom emits the extra energy it possess in the form of light photons. Normally this occurs randomly. Tthe radiation is emitted in all directions and is incoherent. The emission of light from ordinary sources is due to this process. In this decay of atoms, from the upper energy state E_2, to the lower energy state E_1, the energy ΔE , of the emitted light photon is given by,

$$\Delta E = E_2 - E_1 = hf$$

where h is the Planck constant and f is the frequency of the emitted photon.

3) Stimulated Emission

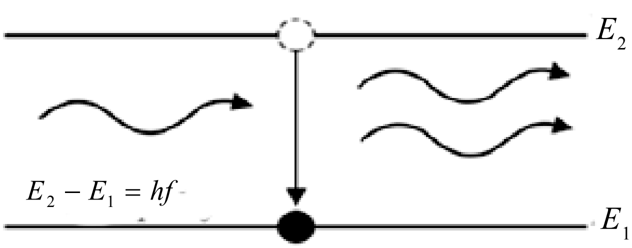


Figure 8.7

However, if a photon of exactly the correct energy approaches an excited atom, an electron in a higher energy level E_1 may be induced to fall to a lower level E_2 and emit another photon. The remarkable fact is that this photon has the same phase, frequency and direction of travel as the stimulating photon which is itself unaffected. This energy is again given by,

$$\Delta E = E_2 - E_1 = hf$$

where h is the Planck constant and f is the frequency of the emitted photon.

In a laser, it is arranged that light emission by stimulated emission exceeds that by spontaneous emission. To achieve this it is necessary to have more electrons in an upper than a lower level. Such a condition, called an 'inverted population', is the reverse of the normal state to affairs but it is essential for light amplification, i.e. for a beam of light to increase in intensity as it passes through a material rather than to decrease as is usually the case.

One method of creating an inverted population is known as 'optical pumping' and consists of illuminating the laser material with light. Consider two levels of energies E_1 and E_2, where $E_2 > E_1$. If the pumping radiation contains photons of frequency $(E_2 - E_1)/h$, electrons will be raised from level 1 to level 2 by photon absorption. Unfortunately, however, as soon as the electron population in level 2 starts to increase, the pumping radiation induces stimulated emission from level 2 to level 1, since it is of the correct frequency and no build up occurs.

In a three level system, Figure 8.8, the pumping radiation of frequency $(E_3 - E_1)/h$, raises electrons from level 1 to level 3, from which they fall by spontaneous emission to level 2. An inverted population can arise between level 2 and 1 if electrons remain long enough in level 2. The spontaneous emission of a photon due to an electronic fall from level 2 to level 1 may subsequently cause the stimulated emission of a photon which in turn releases more photons from other atoms. The laser action thus occurs between level 2 and 1 and the pumping radiation has different frequency from that of the stimulated radiation.

Practical Laser

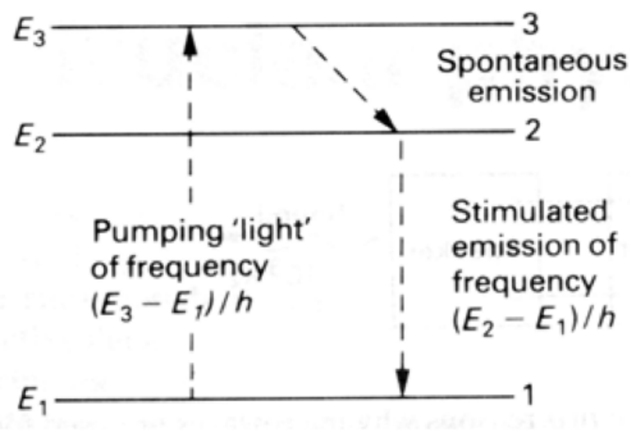


Figure 8.8 - Action of Laser in a three level system

A practical laser set-up consists of three main components as shown below in Figure 8.9.

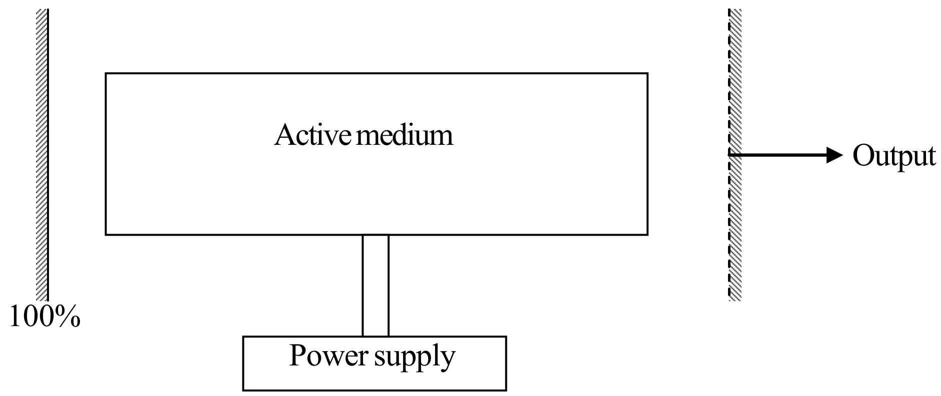


Figure - 8.9

Basically, it needs an Active Medium or a Material Medium with the required atoms (or molecules) to generate a photon beam, then a Power Source to provide energy to excite the atoms (or molecules) in the medium to higher energy levels and a resonator to provide feedback to build up a stream of coherent photons from this experimental arrangement. The resonator comprises two mirrors, one of which is usually highly reflecting (can extend even up to 100 %) and the other slightly less reflective (still up to about 90 to 95%).

In a practical, laser initially the power source is switched on, which in turn provide energy to the medium. This energy will pump the ground state atoms in the medium to their excited states through t he process of absorption. This is illustrated in Figure 8.10(a), which is shown above. When there are a large number of atoms in their excited states they tend to interact with photons produced by the energy source, which in turn dislodge the excited atoms and force them to fall back to their ground state emitting photons, through stimulated emission, which are in phase with the stimulating photons that force the excited atoms to emit photons as depicted in Figure 8.10 (b).

While this process takes place, a small percentage of the excited atoms would also tend to emit the extra energy in the form of photons through spontaneous emission and get back to their ground state. Then there are also photons that are emitted through stimulated emission by photons emitted by excited atoms but travelling at an inclination to the axis. All these photons emitted at an angle to the axis of the system do not contribute to the main beam but are lost from the beam.

If the photons emitted through stimulated emission and the stimulating photons that are all in phase travel along the axis, then they interact further with excited atoms that lie on their path building up a stream of photons all in phase with each other, until they are incident on one of the resonator end mirrors [see Figure. 8.10 (a)]. At the end mirror, the beam of photons is reflected back into the active medium and will again travel along the axis, all in phase with each other.

This process of photons travelling back and forth between the two resonator mirrors will go on, traversing the medium several times, stimulating more and more excited atoms to emit photons, all lying in phase with the stimulating photons, until the intensity is sufficiently high for it to leave the partially reflecting mirror as a highly powerful photon beam where all photons are in phase with each other [Figure 8.10 (c)].

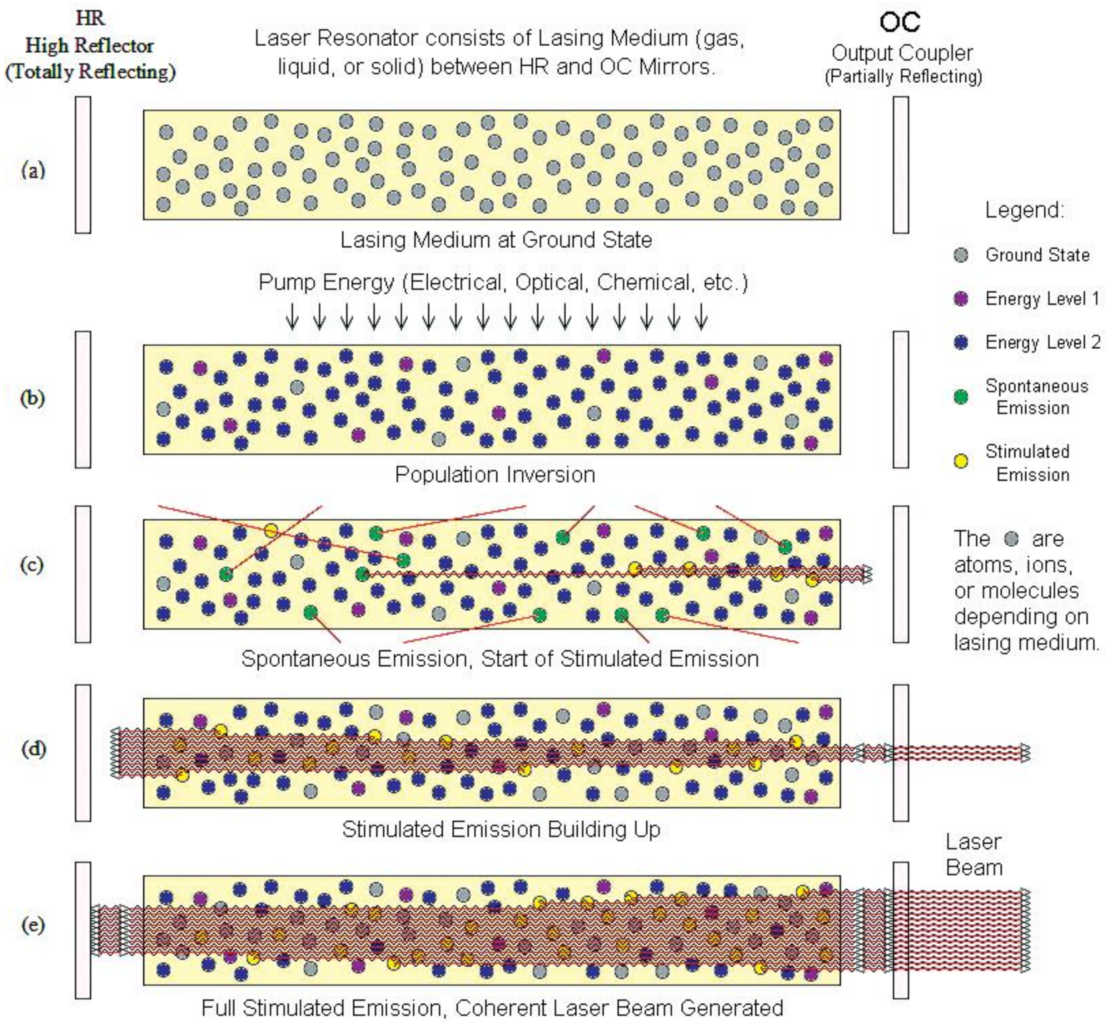


Figure 8.10

Ruby Laser

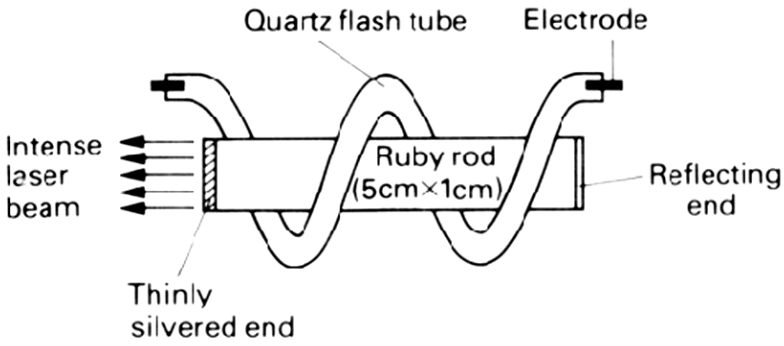


Figure 8.11 Ruby laser

The appearance of the Ruby Laser heralded the Laser age in Optics and aroused a flurry of Activity in this field. Many materials can be used in laser. The ruby rod laser consists of a synthetic crystal of aluminium oxide containing a small amount of chromium as the laser material. It is a type of three-level leaser in which 'level'3 consists of a band of very close energy levels. The pumping radiation, produced by intense flashes of yellow-green light from a flash tube, Figure 8.11, raises electrons from level 1 (the ground level) into one of the levels of the band. From there they fall spontaneously to the metastable level 2 where they can remain for approximately 1 millisecond, as compared with 10^{-8} second in the energy band. Red laser light is emitted when they are stimulated to fall to level 1 from 2. One end of the ruby rod is silvered to act as a complete reflector whilst the other is thinly silvered and allow partial transmission. Stimulated light photons are reflected to and fro along the rod producing an intense beam, part of which emerges from the partially silvered end as the useful output of the laser.

Helium – neon laser

By the end of 1960, Ali Javan and co-workers at the Bell Telephone Laboratories in Holmdel, New Jersey, generated coherent radiation from a gas discharge consisting of Helium and Neon. This uses a mixture of helium and neon, and whereas the ruby laser emits short pulses of light, it works continuously and produces a less divergent beam. In one form the gas is in a long quartz tube with an optically flat mirror at each end. Pumping is done by a 28 M Hz r.f. generator instead of a flash tube. An electric discharge in the gas pumps the helium atoms to an excited state. They then excite the neon atoms to a higher state by collision and produce an inverted population of neon atom which emit radiation when they are stimulated to fall to a lower level.

Applications of the Laser

Lasers in Industry

Laser systems covering a wide frequency spectrum of varying output power capabilities are already in the market. Industrial laboratories the world over are working on high energy devices for applications involving drilling, cutting, scribing, welding and semiconductor manufacturing. Each of these applications require unique parameters in terms of energy, power, wavelength, the beam profile and the like, and it's the $10\ \mu m$ Carbon Dioxide Laser that dominates as the widely used system. Its solid state counterpart is the Nd YAG laser generating coherent light at $1.06\ \mu m$ .This shorter wavelength is more readily absorbed by metals making it more desirable in electronic soldering, spot-welding and micro-hole burning.

A machine that may well invade the industrial establishments is the laser robot combination called the "Combo" that is presently being used in several enterprises. Another device that has been produced recently entitled "Makerarm" is a good example of such a laser robot- combination. It is an all-in-one Robotic Laser Cutter, 3D Printer, Painter, Fabricator and an Assembler.

Lasers in Medicine

Instruments consisting of highly stable laser oscillators are used by surgeons in practically every aspect of the healing arts, one of the earliest applications being that of dealing with detached retinas of the human eye. Reattachment is effected by means of a low energy Ruby laser, where the retina is spot welded back in place while leaving the tissues unaffected only a few cell widths away.

Lasers are one of the most effective means of dealing with surgery involving the brain and the spinal cord. A laser beam can serve as a scalpel, and the immediate cauterization prevents excessive bleeding.

The process termed 'photo-radiation' therapy combines laser light with a cancer tissue sensitizer, the sensitizer producing highly reactive chemicals to destroy the host cancer cells when exposed to red Argon laser light. The technique also has applications in the diagnosis of cancer, where a different fluorescent dye and a blue light emitting Frypton laser scanner are employed.

Chapter - 09

Geometrical optics

Refraction of light

Velocity of light in a vacuum or free space is considered to be 3×10^8 m s^{-1} approximately. However when light passes through other transparent media such as glass and water, there are changes in the velocity of light in the media accompanied by changes in wavelengths too. These changes cause deviations of the rays of light when passing through such media. This deviation or change in the direction of light rays when passing from one transparent medium to another is referred to as refraction of light.

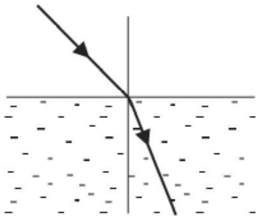


Figure 9.1

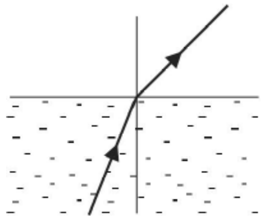


Figure 9.2

When light passes from air to a medium such as glass, its velocity decreases and the ray gets refracted or deviated towards the normal drawn to the interface at the point of incidence (Figure 9.1).

The medium in which the speed of light is slower than that in the other is referred to as an optically dense medium. Thus glass is a dense medium relative to air.

On the other hand, when light passed from a dense medium to a rare medium it gets refracted or deviated away from this normal.

However, if the incident ray is along the normal the ray passes along the normal without any deviation.

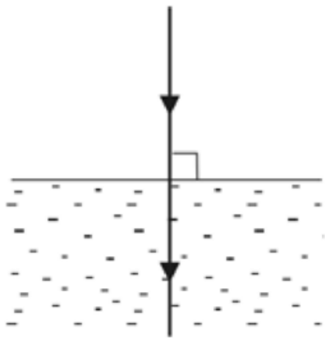


Figure 9.3

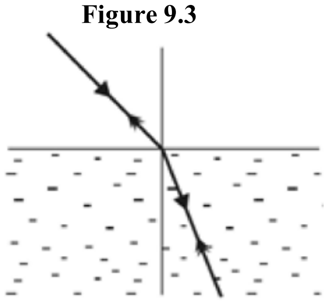


Figure 9.4

Also when a refracted ray is reversed, it re-traces its original path.

Laws of refraction

(1) The incident ray, the refracted ray and the normal to the interface at the point of incidence all lie on the same plane.

(2) For all rays of light of a given colour passing through two given media, the sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction.

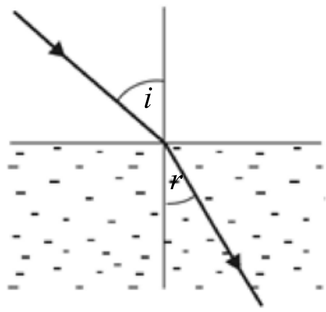


Figure 9.5

$$\frac{\sin i}{\sin r} = n \text{ (a constant)}$$

(1) If the first medium (a) is a vacuum or air the above ratio is known as the "absolute refractive index" of the second medium.

(2) If the first medium is any other than a vacuum or air, then the above ratio is called the refractive index of the second medium relative to the first, ${}_{a}n_{b} = \frac{\sin i}{\sin r}$

Refractive index can also be expressed in terms of the velocities of light in the two media as well as their wave lengths.

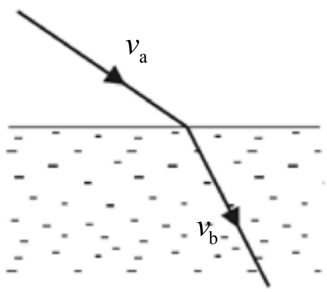


Figure 9.6

If v_a and v_a are the velocities of light of the two media (a) and (b) respectively, then,

$${}_{a}n_{b} = \frac{v_a}{v_b} = \frac{f\lambda_a}{f\lambda_b} = \frac{\lambda_a}{\lambda_b}$$

Considering a ray passing from medium (b) to medium (a) as shown in the figure drawn above,

$${}_{b}n_{a} = \frac{V_b}{V_a}$$

$$\therefore {}_{b}n_{a} = \frac{1}{{}_{b}n_{a}}$$

For example ${}_{a}n_{g} = \frac{3}{2}$; then ${}_{g}n_{a} = \frac{1}{3/2} = \frac{2}{3}$

Also considering light passing through four given media a, b, c and a again, as shown in Figure 9.7

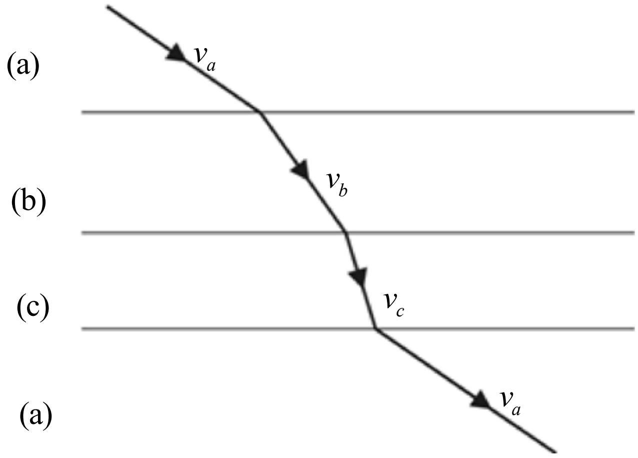


Figure 9.7

$$_{a}n_{b} = \frac{v_a}{v_b}$$

$$_{b}n_{c} = \frac{v_b}{v_c}$$

$$_{c}n_{a} = \frac{v_c}{v_a}$$

$$\therefore \ _{a}n_{b} \times {_{b}n_{c}} \times {_{c}n_{a}} = \frac{v_a}{v_b} \times \frac{v_b}{v_c} \times \frac{v_c}{v_a} = 1$$

Hence, $_{b}n_{c} = \dfrac{1}{_{a}n_{b} \times {_{c}n_{a}}} = \dfrac{_{a}n_{c}}{_{a}n_{b}}$

$$_{b}n_{c} = \frac{_{a}n_{c}}{_{a}n_{b}}$$

For example. $_{a}n_{g} = 3/2, \ _{a}n_{w} = 4/3$

$$\therefore \ _{w}n_{g} = \frac{_{a}n_{g}}{_{a}n_{w}} = \frac{3/2}{4/3} = 9/8$$

Image formed by reflection

O is an object situated in a dense medium such as glass. A ray OA from O passes along AB, and another ray OP normal to the interface passes along PQ undeviated when entering a rare medium above such as air. To an eye receiving the reflected rays PQ and AB, object O appears to be at I. I is the image of object O (Figurer 9.8).

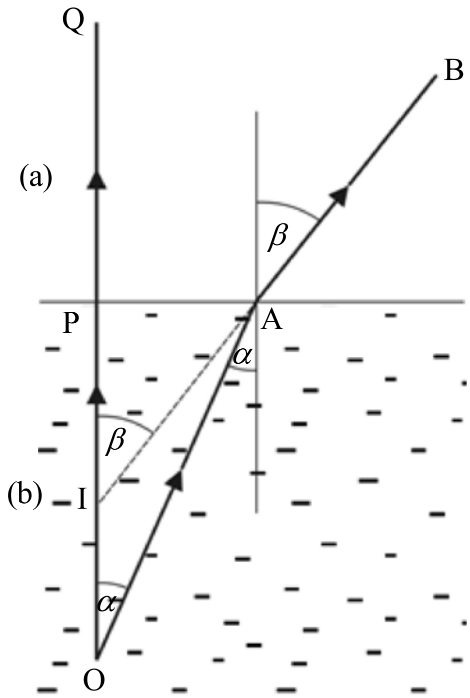


Figure 9.8

For light passing from (b) to (a), $_{b}n_{a} = \dfrac{\sin\alpha}{\sin\beta}$

$$\therefore \ _{a}n_{b} = \frac{1}{_{b}n_{a}} = \frac{\sin\beta}{\sin\alpha} = \frac{PA/IA}{PA/OA} = \frac{OA}{IA}$$

To an eye looking vertically from above P, A is very close to P.

$\therefore \text{OA} \approx \text{OP}$ and $I\text{A} \approx \text{IP}$

$$\therefore {}_a n_b = \frac{\text{OP}}{\text{IP}} = \frac{Real\ depth}{Apparent\ depth}$$

Apparent displacement

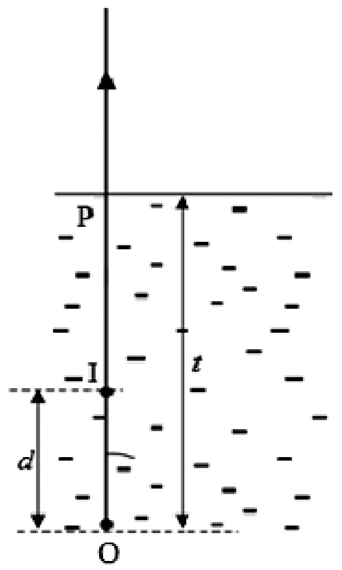


Figure 9.9

The object O appears to have displaced to I due to refraction

apparent displacement (d) = OI = OP- IP

$$d = \text{OP} - \frac{\text{OP}}{{}_a n_b} = \text{OP}\left(1 - \frac{1}{{}_a n_b}\right)$$

$$d = t\left(1 - \frac{1}{{}_a n_b}\right)$$

The above displacement is also valid for a parallel sided transparent block of thickness t, for an object below it,

whatever its position below the block is.

Total internal reflection

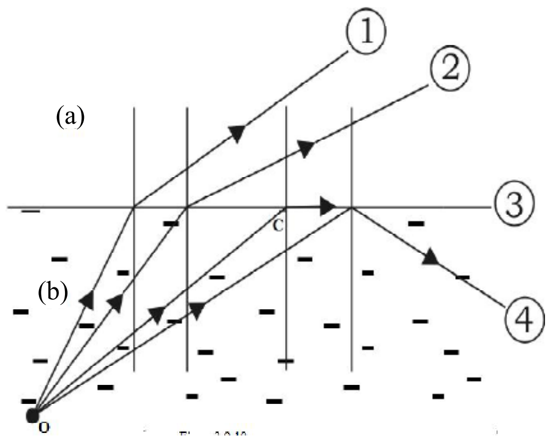


Figure 9.10

Consider light passing from a dense medium (b) such as water or glass to a rare medium (a) such as air.

(1) On entering the rare medium, the ray bends away from the normal towards the interface.

(2) On increasing the angle of incidence, the refracted ray bends more towards the interface.

(3) On further increasing the angle of incidence, a situation is reached when the ray appears to pass along the interface separating the two media. The angle of incidence at this stage is called the "critical angle".

(4) When the angle of incidence exceeds the critical angle, the interface acting as a mirror reflects the ray completely back into the dense medium. This phenomenon is called 'total internal reflection'.

(It should be known that a certain amount of partial reflection takes place throughout, before total internal reflection begins to occur)

Conditions for total internal reflection

Light should pass from a dense medium to a less dense (or rare) medium.

The angle of incidence should exceed the critical angle.

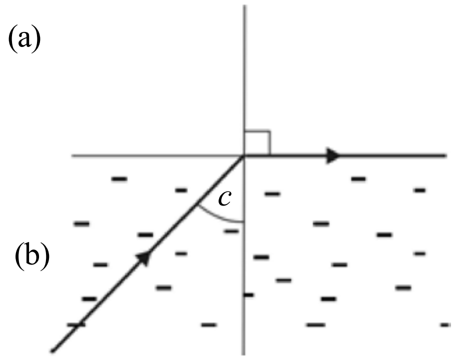


Figure 9.11

Considering the state of the critical angle, since light is travelling from a dense medium (b) to a rare (or less dense) medium (a),

$$_{b}n_{a} = \frac{\sin c}{\sin 90^{o}} = \frac{\sin c}{1}$$

$$\therefore\ _{a}n_{b} = \frac{1}{\sin c}$$

Worked examples

(1) The bottom of a tank is a glass slab of thickness 6 cm and refractive index $3/2$. The tank contains clear water to a depth of 8 cm, the refractive index of water being $4/3$. What is the apparent displacement of a mark on the lower surface of the glass bottom as seen by an observer looking from above the tank in air?

Solution

Since the observer is looking from air,

the apparent displacement due to the glass slab $= 6\left(1-\frac{1}{3/2}\right) = 6\times\frac{1}{3} = 2\text{ cm}$

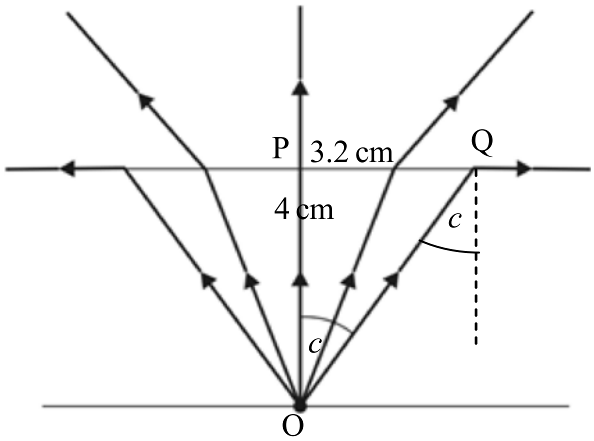
The apparent displacement due to water $= 8\left(1-\frac{1}{4/3}\right) = 8\times\frac{1}{4} = 2\text{ cm}$

$\therefore$ Total apparent displacement $= 2+2 \quad = 4\text{ cm}$

Worked Example

A rectangular slab of glass 4.0 cm thick has a luminous point on its lower surface. Light from this point strike the upper surface of the glass slab and emerge from it illuminating the surface until the rays strike the surface at the critical angle, after which the rays undergo total internal reflection outlining an illuminated circular patch of radius 3.2 cm. What is the critical angle of the glass? Hence find the refractive index of the glass.

Solution



If c is the critical angle of glass,

$$\tan c = \frac{3.2}{4.0} = \frac{32}{40} = \frac{4}{5} = 0.8000$$

$$\therefore c = \tan^{-1}\left(0.8000\right) = 38^{\circ}40'$$

$\therefore$ Refractive index of glass,

$$n = \frac{1}{\sin c} = \frac{1}{\sin\left(38^{\circ}40'\right)} = 1.60$$

Refraction through prisms

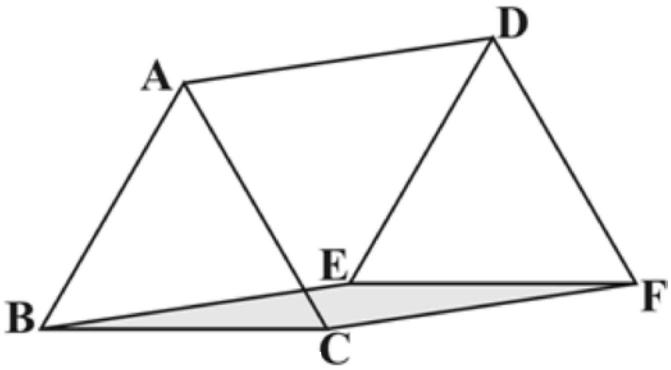


Figure 9.12

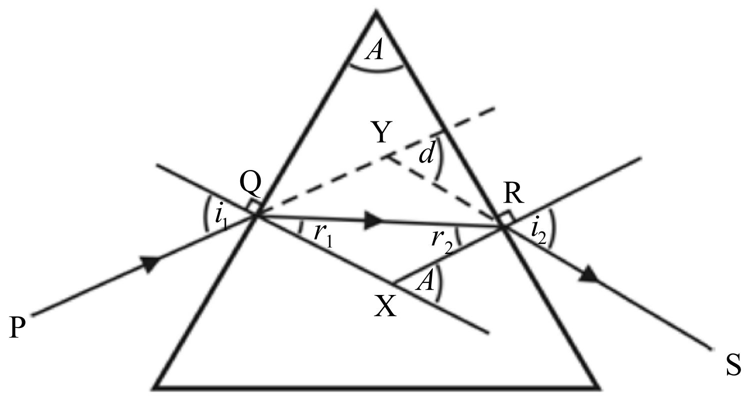


Figure 9.13

A prism is bounded by two triangular faces and three rectangular faces. The triangular section such as ABC is called a principal section of the prism and its vertex angle A, the refracting angle or the prism angle.

PQRS is the path of a ray of light passing through a prism of a transparent medium such as glass. At Q, the ray deviated towards the normal when entering the dense medium while at R it deviates away from the normal when leaving the dense medium to enter the rare medium. The angle '*d*' between the incident ray PQ and the emergent ray RS is the angle of deviation.

If i_1, i_2, r_1 and r_2 are the angles as shown and *A* is the refracting angle of the prism, then by using geometry it can be shown that,

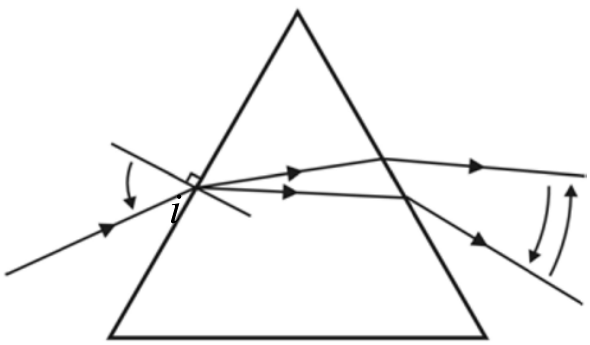
$$r_1 + r_2 = A \quad \text{.............................9.1}$$

$$(i_1 - r_1) + (i_2 - r_2) = d$$

$$(i_1 + i_2) - (r_1 + r_2) = d$$

$$i_1 + i_2 - A = d$$

$$\therefore\ i_1 + i_2 = A + d \quad \text{......................9.2}$$



Figer 9.14

It can be shown that, when the angle of incidence is increased starting from a small value, the emergent ray first moves in such a direction as to decrease the angle of deviation. Then it reaches a position where the deviation is minimum and then turns back to move in the direction of increasing the angle of deviation.

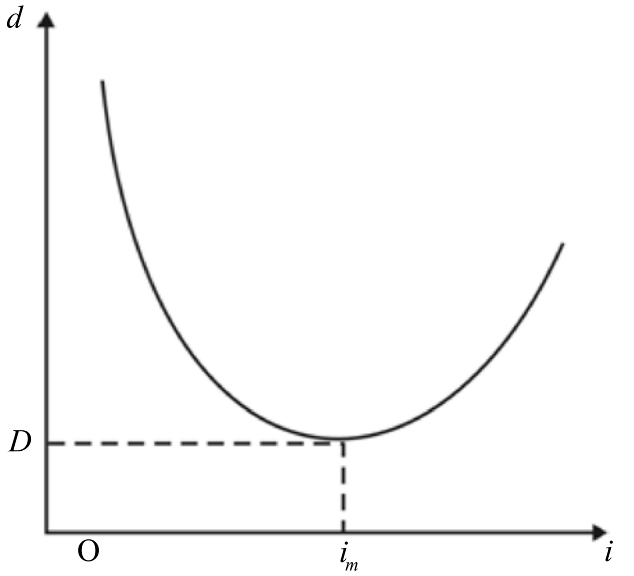


Figure 9.15

If a graph,is plotted, deviation (*d*) against the angle of incidence (*i*), a curve as illustrated in Figure 9.15 is obtained.

The graph shows that there is an angle of minimum deviation (*D*).

At the state of minimum deviation the ray is found to pass symmetrically through the prism.

That is, in the position of minimum deviation,

$i_1 = i_2 = i$ and $r_1 = r_2 = r$

$$\therefore 2r = A$$
$$r = A/2$$

also $2i = A + D$

$$i = \frac{A+D}{2}$$

$\therefore$ Refractive index of the material of the prism,

$$n = \frac{\sin i}{\sin r}$$

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\frac{A}{2}}$$

Worked Example

Calculate the angle of minimum deviation of a prism of refracting angle 60° and made of glass of refractive index 1.50.

Solution

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\frac{A}{2}}$$

$$1.50 = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\frac{60}{2}} = \frac{\sin\left(\frac{A+D}{2}\right)}{0.5}$$

$$\sin\left(\frac{A+D}{2}\right) = 1.50 \times 0.5 = 0.75$$

$$\frac{A+D}{2} = \sin^{-1}(0.75) = 48^\circ 35'$$

$$D = 2 \times 48^\circ 35' - A = 97^\circ 10' - 60^\circ = 37^\circ 10'$$

Deviation of light rays by a glass prism

Taking refraction index of glass as approximately $n = 1.50$, the critical angle is,

$$c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.5}\right) = \sin^{-1}(0.667) = 42^\circ$$

Hence a ray of light striking a glass-air interface from glass at an angle of incidence greater than 42o undergo total internal reflection. Making use of this fact, a right angled isosceles glass prism can be used to perform following deviations.

Lateral inversion occurs in each case

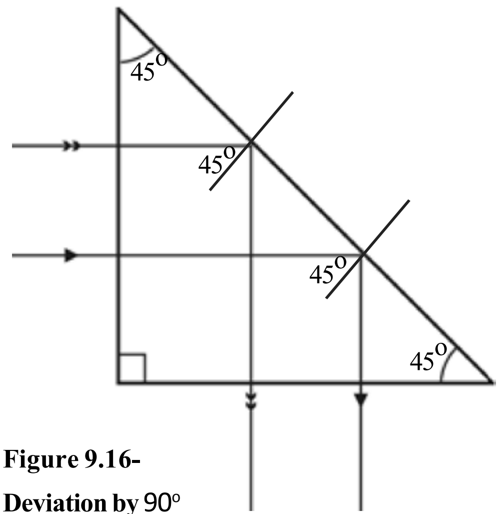


Figure 9.16- Deviation by 90°

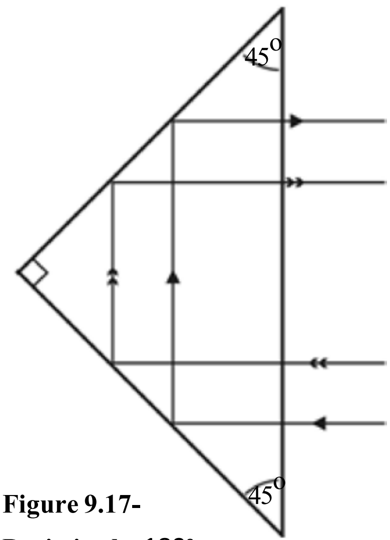


Figure 9.17- Deviation by 180°

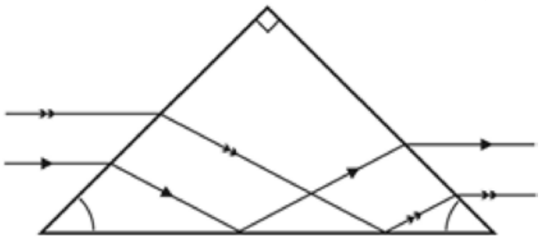


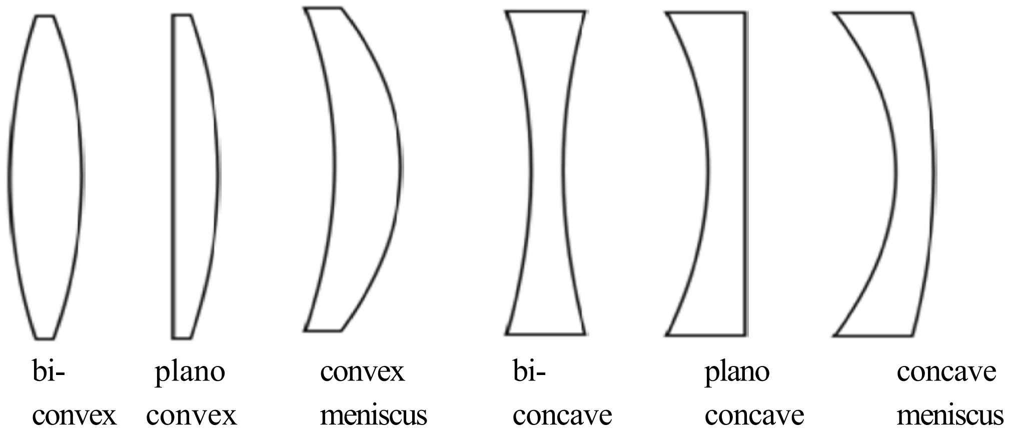
Figure 9.18-Deviation by 360°

Refraction through thin lenses

Lenses are divided into two categories mainly on their geometrical shapes, convex lenses being thicker in the middle and thinner at the edges while concave lenses are thinner in the middle than at the edges.

All convex lenses converge beams of light passing through them and are hence also called converging lenses while all concave lenses which diverge beams passing through them are called diverging lenses.

The midpoint of a lens through which the rays are supposed to pass without any deviation is called the "optical centre" of the lens and the line joining the centres of curvature of the two lens surfaces is the "principal axis" of the lens.



Figur 9.19

Principal foci

Rays parallel and close to the principal axis of a convex lens converge to a common point F_2 on the principal axis (Figure 9.20).

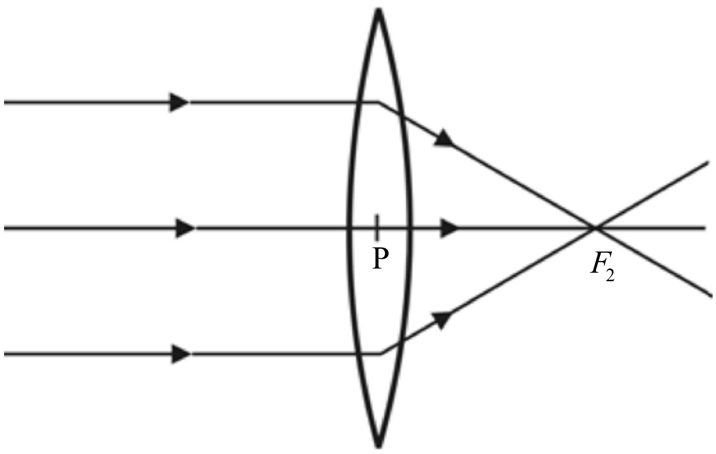
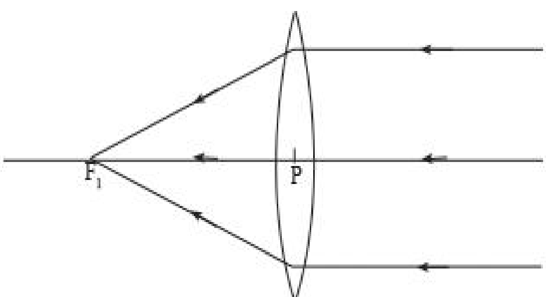


Figure 9.20

A similar beam from the opposite side of the lens converge to a common point F_1 on the other side (Figure 9.21)



Figurer.9.21

These points F_1 and F_2 are referred to as the principal foci of the lens.

Similarly, principal foci exist for concave lenses as follows.

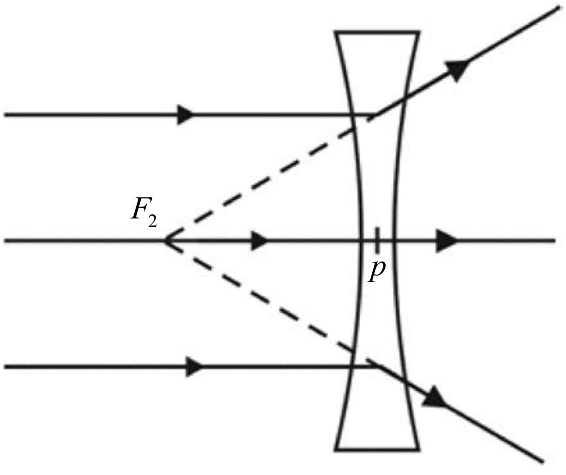


Figure 9.22

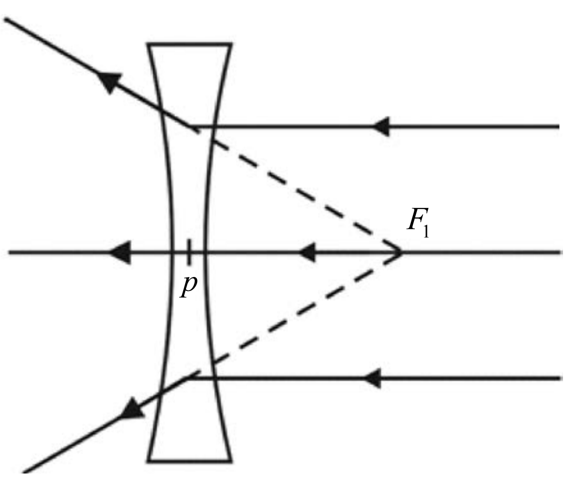


Figure 9.23

The distance from the optical centre to each principal focus is called the focal length of the lens. For any lens the two focal lengths are equal to each other.

Focal planes

Planes passing through the focal points of a lens normal to the principal axis are called focal planes.

The parallel beams which are not parallel to the principal axis will,

(1) Converge to a point on the focal plane on the opposite side in case of a convex lens.

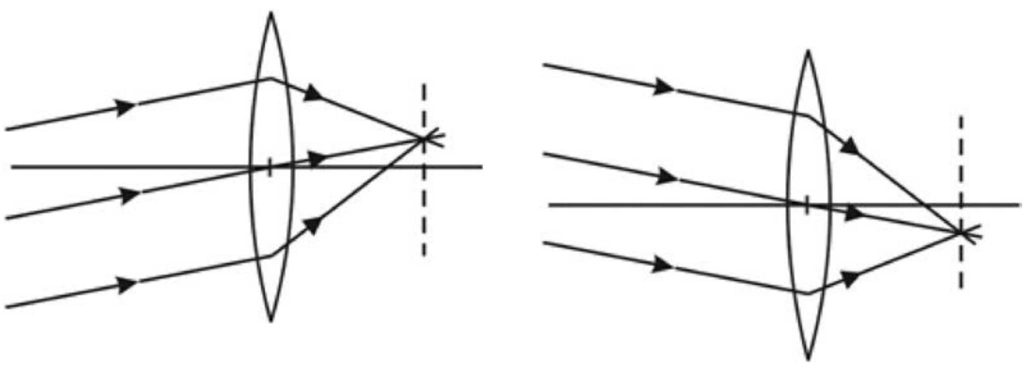


Figure 9.24

(2) Appear to diverge from a point on the focal plane on the same side in case of a convex lens.

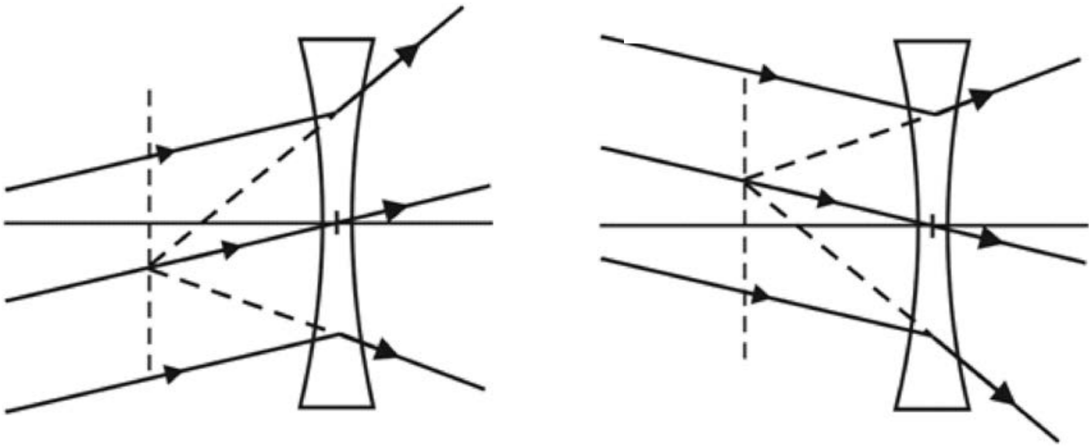


Figure 9.25

Formation of images by lenses

Images formed by lenses can be constructed by using two rays from the topmost point of the object, one passing parallel to the axis of the lens and the other passing through the optical centre of the lens and then locating the point where the rays meet after refraction through the lens.

Convex lenses

1. Object at infinity

<u>Image</u>

Formed at *F*.

Real and almost a point image.

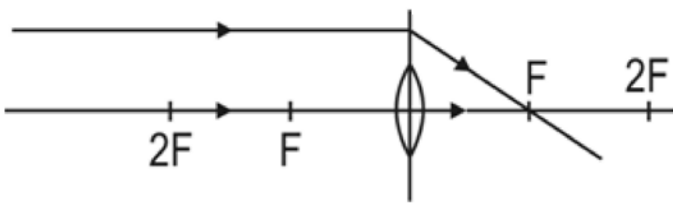


Figure 9.26

2. Object beyond *2F*

Formed beyond *F* on opposite side

Real, diminished and inverted

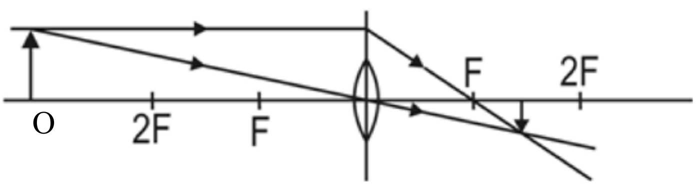


Figure 9.27

3. Object on $2F$

Formed on $2F$ on opposite side

Real, inverted and equal in size

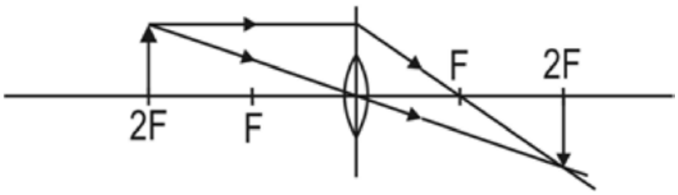


Figure 9.28

4. Object between $2F$ and F

Formed beyond $2F$ on opposite side

Real, inverted and enlarged

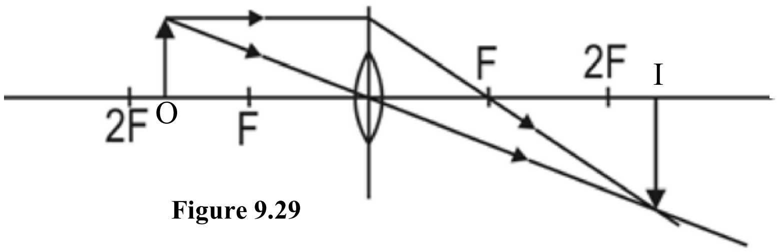


Figure 9.29

5. Object on F

Formed at infinity

Enlarged

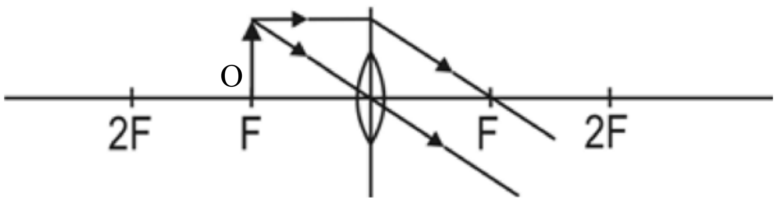


Figure 9.30

6. Object between F and $2P$

Virtual, upright and enlarged

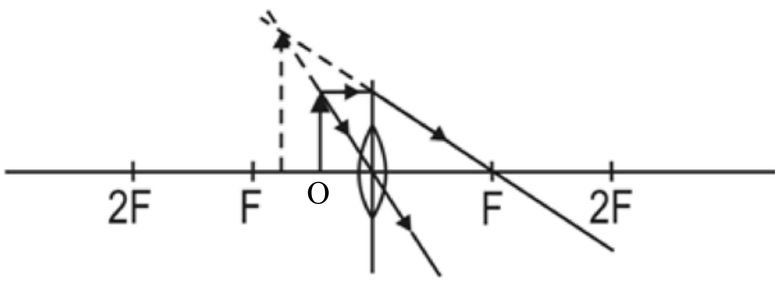


Figure 9.31

All real images formed by convex lenses can be interchanged with their objects. Such points where the object and the corresponding image are interchangeable are called “conjugate points”.

Images formed by concave lenses

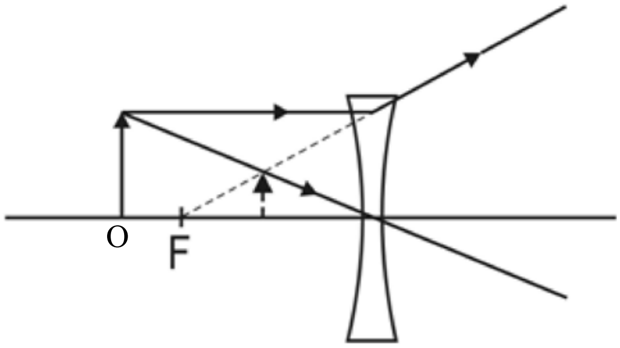


Figure 9.32

For all positions of the object the image is virtual, diminished, upright and forms between the focus and the lens.

Location of real images formed by convex lenses

The method of no-parallax is used in the location of real images conveniently.

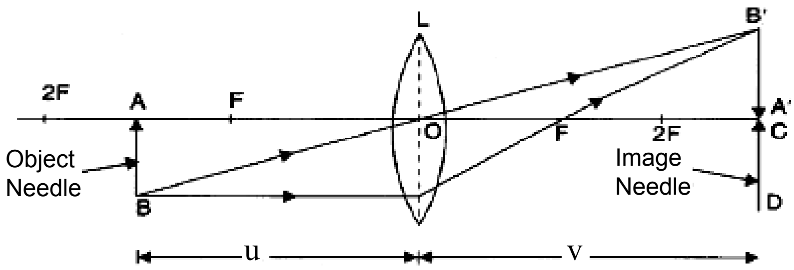


Figure 9.33

The lens L and the object pin O are placed along the same axis to form the real image *I* which will be inverted.

In order to locate the position of this image, a locating pin O_1 is adjusted until there is no-parallax between O_1 and I. This is achieved by moving the observer’s eye sideways while adjusting the pin O until the tips of image *I* and locating pin O_1 appear to coincide and move together without any relative motion.

After locating the position of the image, image distance and object distance can be measured and used for any calculations involving object distance, image distance and focal lengths.

The lens formula

The lens formula, a relation connecting the object distance and the image distance with the focal length of a lens can be derived using any of the ray diagrams containing both the object and the image.

Eg:-

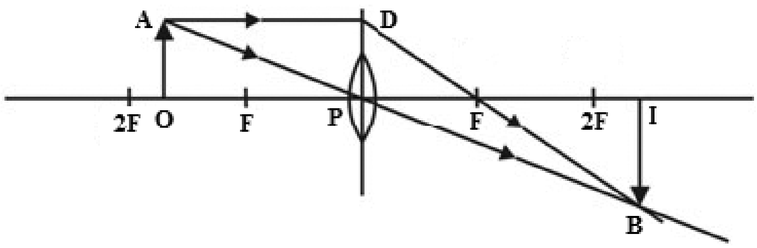


Figure 9.34

From similar triangles IBF and OAP,

$$\frac{IB}{OA} = \frac{IP}{OP} \text{.......................} 9.3$$

From similar triangles IBF and PDF,

$$\frac{IB}{PD} = \frac{IF}{PF} \text{....................} 9.4$$

Since PD = OA, from equations 9.3 and 9.4

$$\frac{IP}{OP} = \frac{IF}{PF}$$

$$IP \times PF = IF \times OP$$

$$IP \times PF = (IP - PF)OP$$

$$IP \times PF = IP \times OP - PF \times OP$$

$\div IP \cdot PF \cdot OP$ $$\frac{1}{OP} = \frac{1}{IF} - \frac{1}{IP}$$

$$\therefore \frac{1}{OP} + \frac{1}{OP} = \frac{1}{IF} \text{....................} 9.5$$

To obtain a common formula for both types of lenses and also for both types of images (real and virtual) a sign convention is applied.

Cartesian sign convention

All distances are measured from the optical centre of the lens. Those distances measured in the direction of incident light are considered negative while those measured against the direction of incident light are considered positive.

According to the above sign connection and referring to the ray diagram,

$\text{IP} = -v; \quad \text{OP} = +u; \quad \text{IF} = -f$

$\therefore$ From (3) $\dfrac{1}{-v} + \dfrac{1}{u} = \dfrac{1}{-f}$

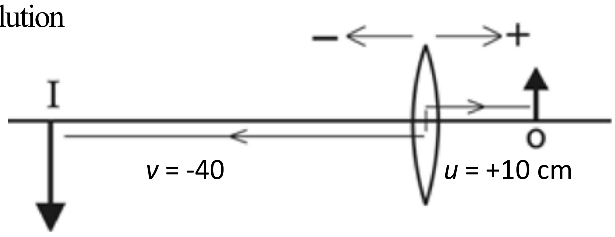
$\therefore \quad \dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$ is the lens formula.

Also linear magnification $(m) = \dfrac{height\ of\ image}{Height\ of\ object} = \dfrac{\text{OP}}{\text{IP}} = \left|\dfrac{v}{u}\right|$

Worked examples

(1) An object placed 10 cm in front of a convex lens forms a real image of magnification 4. What is the focal length of the lens? Where should the object be placed in order to form a virtual image of the same magnification 4?

Solution



Magnification $(m) = \dfrac{v}{u} = 4$

$= \dfrac{v}{10} = 4 \Rightarrow v = 40 \text{ cm}$

$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\dfrac{1}{-40} - \dfrac{1}{10} = \dfrac{1}{f}$

$\dfrac{-5}{40} = \dfrac{1}{f}$

$f = \text{-8 cm}$

For vertical image : $\frac{v}{u} = 4 \Rightarrow v = 4u$

Since the image is virtual, it is on the same side as the object.

$$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

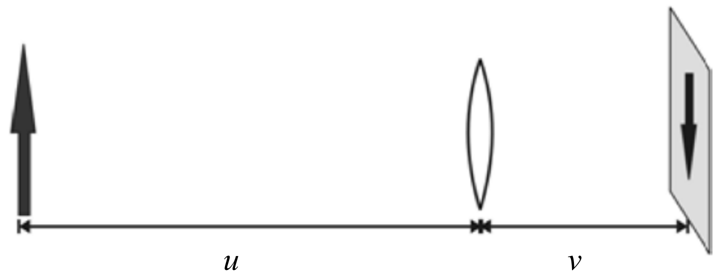
$$\frac{1}{4u} - \frac{1}{u} = \frac{1}{-8}$$

$$\frac{-3}{+4u} = \frac{-1}{8}$$

$$u = \frac{+3 \times 8}{4} = \underline{\underline{6 \text{ cm}}}$$

(2) A small luminous object placed on the axis of a convex lens forms a sharp image of magnification 2 on the screen. The screen was then moved away a distance of 20 cm from the lens and the object was adjusted to produce an image of magnification 3 on the screen. Find the focal length of the lens.

Solution



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Applying sign convection for all quantities,

$$\frac{1}{-v} - \frac{1}{u} = \frac{1}{-f}$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$\times\ v$ $$1 + \frac{v}{u} = \frac{v}{f}$$

$$1 + m = \frac{v}{f}$$

First image : $1 + 2 = \frac{v_1}{f}$ ....(1)

Second image : $1 + 3 = \frac{v_2}{f}$ ....(2)

(1) -(2) gives $1 = \frac{v_2 - v_1}{f} = \frac{20}{f}$

$f = 20$ cm

Lens combinations

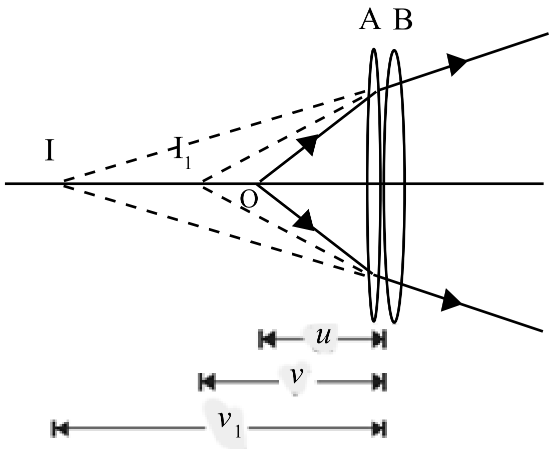


Figure 9.35

Consider two thin lenses A and B of focal lengths f_1 and f_2 respectively placed in contact with each other as shown. Suppose O is a small object on the common axis of the two lenses. Rays from O after refraction through lens A would appear to diverge from point I_1, which should be the image if the lens B was absent.

Applying the lens formula for this refraction,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \text{.......................} 9.6$$

However, due to the lens *B*, the rays from A get refracted through *B* to form the final image *I* for which the first image I_1 can be considered as the object. Hence considering v_1 as object distance for second refraction, and *v* the final image distance,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \text{..................} 9.7$$

9.6 + 9.7 gives,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \text{................} 9.8$$

If this lens combination is equivalent to a single lens of focal length *f*,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{.....................9.9}$$

From 9.8 and 9.9, $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$

The above equation which gives the focal length of a lens combination, is applicable to both types of lenses under respective signs according to the sign convention.

Worked Example

A convex lens of focal length 30 cm is in contact with a concave lens of focal lens 45 cm. What is the focal length of the combination? Is it converging or diverging?

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{-30}+\frac{1}{45}=\frac{-3+2}{90}=\frac{-1}{90}$$

$$\therefore f=-90 \text{ cm}$$

Since the sign is negative the lens combination acts as a converging lens. This gives the impression that the convex lens is more powerful than the concave lens although the focal length of the latter is greater. Hence it can be considered that it is not the focal length '*f*' but its reciprocal $\frac{1}{f}$ which decides how powerful a lens is.

$\therefore$ Power of a lens $P=\frac{1}{f}$

Unit: dioptre (*D*) which is the power of a lens of focal length 1 m.

Sign convention :The power of a convex lens is positive while that of a concave lens is negative. Hence according to the Cartesian sign convention the power can be expressed as ,

Convex lens : +*D*

Concave lens: –*D*

The human eye

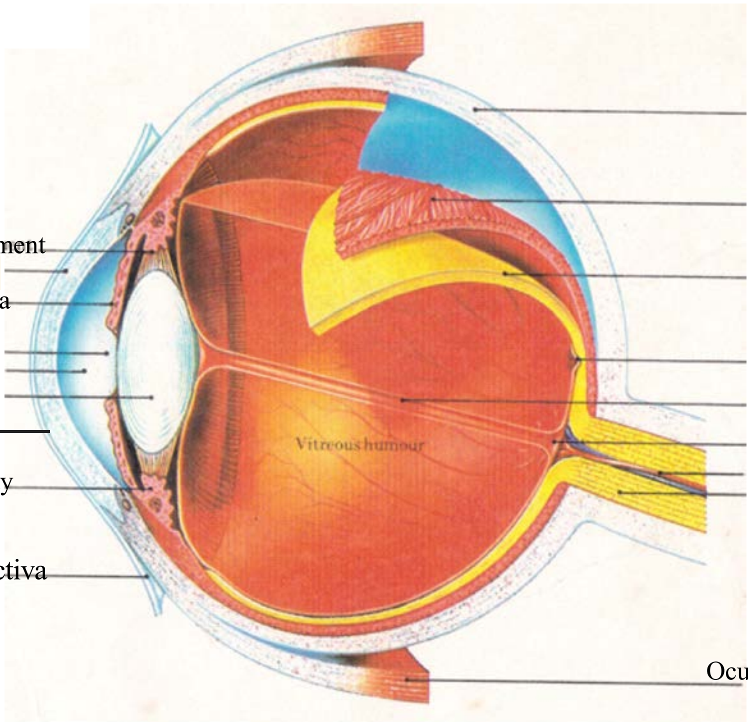


Figure 10.1

The eye is a part of the human body and it provides the sense of vision.

As shown above, the main component of the eye is the eye lens which is a converging lens. It is made up of a transparent jelly like crystalline substance and it is responsible for forming the images of objects seen by it. These images are inverted and are formed on the retina which acts as a screen for the images. The optic nerves containing millions of fibres transmit information from the cells of the retina to the brain.

The pupil is an aperture in front of the lens through which light enters the eye.

The iris is a diaphragm of variable size and adjusts the size of the pupil controlling the amount of the light entering the eye.

The cornea is a strong clear buldge in front of the lens, the gap in between being filled with an aqueous humour which is a watery transparent fluid.

The ciliary muscles controls the shape of the lens to adjust is focal length to suit the image distance between the lens and the retina with varying object distances. This process of either compression or relaxation of the lens is known as accommodation.

Range of distinct vision of an eye

Every eye has a specific range, within which the objects placed can be seen clearly. The point nearest to the eye in this range is called the "near point" of the eye and the distance to it from the eye is the "least distance of distinct vision".

Then, the farthest point of clear vision is the "far point".

For a healthy eye, the near point is at a distance of about 25 cm from the eye while the far point is infinitely away. This is the "range of distinct vision" of a healthy eye. If for a certain eye the near point and the far point differ from the above values, the eye is then said to suffer from defects of vision.

Focusing of image of object at,

(1) Far point

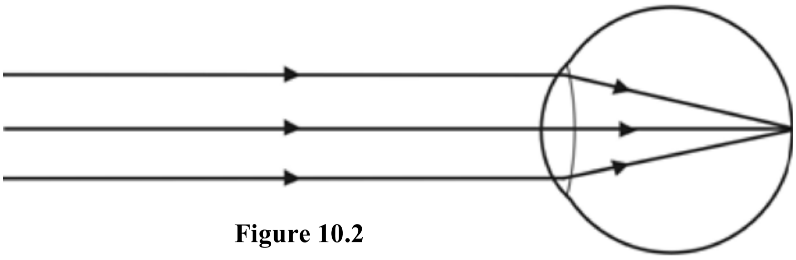


Figure 10.2

(2) Near point by a healthy eye

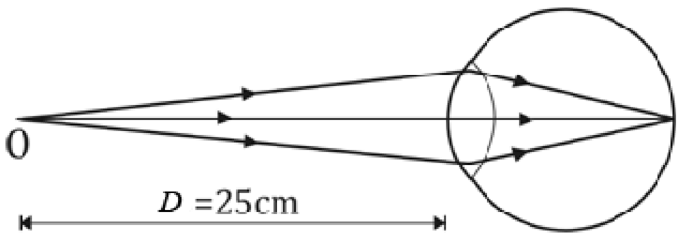


Figure 10.3

Defects of vision

1. Short sight (myopia)

Short sight is the inability to see clearly the objects at infinite distances from the eye. This happens due to the parallel rays from a distant object after refraction through the eye lens focusing to a point in front of the retina, instead of on the retina, thereby not-forming a sharp image on the retina.

Figure 10.4

The remedy is the use of a suitable diverging lens capable of displacing the focusing point further up to the retina.

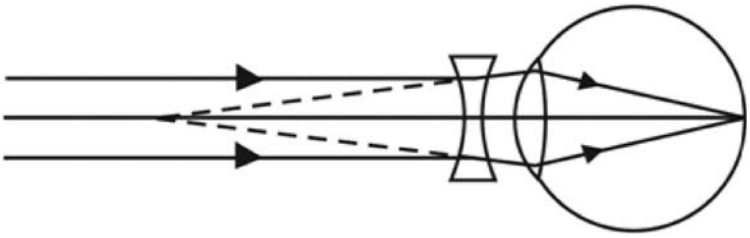


Figure 10.5

For example, consider a short sighted person of whom the far point is at 400 cm from the eye. If he needs to see clearly objects as far away as an infinite distance, a diverging lens capable of forming an image at 400 cm from the eye of an object at infinity has to be selected.

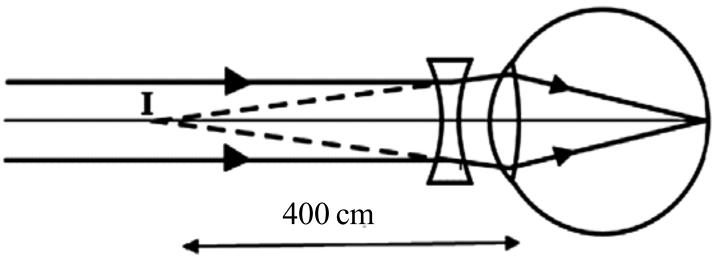


Figure 10.6

$$\text{Using } \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{400} - \frac{1}{\infty} = \frac{1}{f} \Rightarrow f = 400 \text{ cm}$$

A diverging lens of focal length 400 cm has to be selected.

2. Long sight

Long sight is the inability to see clearly the objects at the normal near point of 25 cm. This happens due to the light from the object at 25 cm after refraction through the eye lens converging to focus on a point behind the retina, thereby not forming a sharp image on the retina.

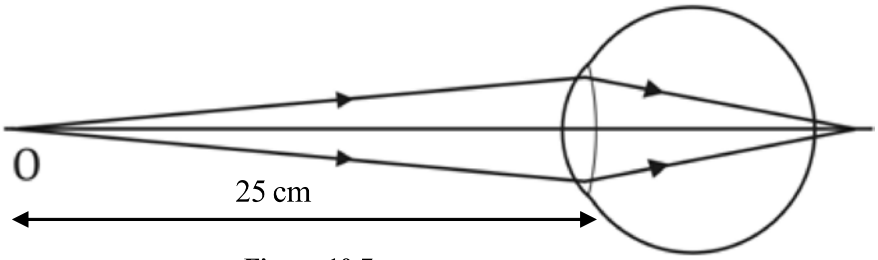


Figure 10.7

The remedy is to use a suitable converging lens capable of shifting the focusing point up to the retina.

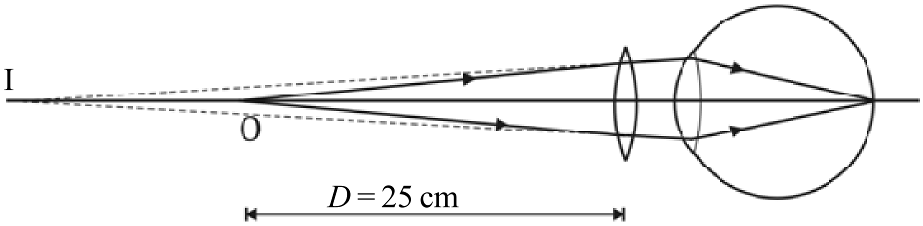


Figure 10.8

For example consider a far sighted person whose near point is at 100 cm from the eye. If he needs to see clearly objects as close as 25 cm from the eye, a converging lens capable of forming an image at 100 cm from the eye of an object placed at 25 cm has to be used.

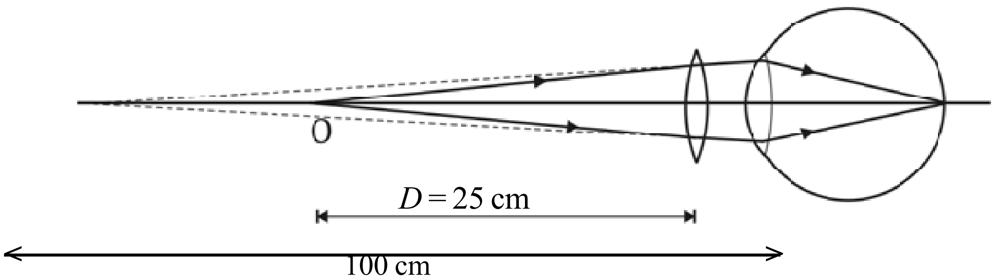


Figure 10.9

Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{100}-\frac{1}{25}=\frac{1}{f} \Rightarrow f=\frac{-100}{3}=-33\ \frac{1}{3}$ cm

A converging lens of focal length $33\frac{1}{3}$ cm has to be used.

3. Presbyopia

This defect is associated with aging in which a person develops a progressively diminished ability to focus clearly on near objects. The initial signs are eye strain, difficulty to see in dimlight, problems in focusing small objects etc. People in the age group of 40-50 years usually develop this defect. This defect is corrected using suitable lenses.

4. Astigmatism

Astigmatism is a defect that arises due to the differences of the curvature in different planes of the refracting surface of the front of the cornea. For example, due to such a difference between the curvatures in the horizontal and vertical planes, there will be no sharp image as the focal lines will be different in the two planes. The eye trying to focus both at the same time causes muscular strain. Astigmatism is corrected by using cylindrical lenses.

Worked Example

The far point of a short sighted person is 100 cm while his near point is 20 cm from the eye.
What lens will he need to see distant objects clearly?
What will be his near point when using this lens?

Solution: In order to see distant objects clearly, an object at an infinite distance should form a virtual image at 100 cm from the eye, when using a correcting lens.

Using $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{100}-\frac{1}{\infty}=\frac{1}{f} \Rightarrow f = 100 \text{ cm}$$

$\therefore$ A diverging lens of focal length 100 cm is needed.

In order to find the near point when wearing the lens,

$u = ?$ $v = 20$ cm $f = 100$ cm

using $$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{20}-\frac{1}{u}=\frac{1}{100} \Rightarrow u = 25 \text{ cm}$$

The near point when using the correcting lens is 25 cm from the eye.

Chapter - 11

Optical instruments

Visual angle

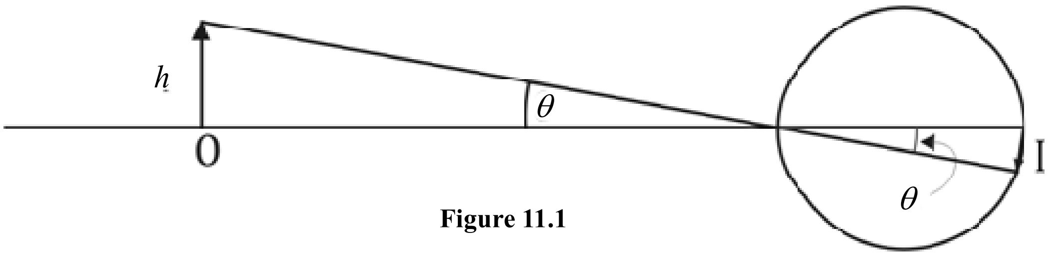


Figure 11.1

Suppose an object *O* is placed in front of an eye and let *I* be the image formed on the retina. The angle θ subtended by the object at the eye is called the "visual angle" and equals the vertically opposite angle θ subtended by the image *I* as shown in the figure 11.1. Hence it is apparent that larger the visual angle, larger is the image on the retina.

Optical instruments are therefore designed to increase the visual angle subtended by objects at the eye by forming images subtending larger visual angles at the eye.

Laying more emphasis on the visual angles than on the size of the image and the object, the magnifications produced by optical instruments are measured by the "angular magnification" instead of linear magnification. The general definition of angular magnification is,

$$m = \frac{\text{Angle subtended at the eye by the image}}{\text{Angle subtended at the eye by the object}}$$

However, depending on the positions of the image and the object in the particular adjustment of the instruments, the expressions in the above definition can get modified accordingly.

Microscopes

Microscopes are those optical instruments that are used to form and view enlarged images of micro-objects located close to the eye. When using microscopes, it is customary for the final image to be formed at the near point (25 cm) of the eye. Then the microscope is said to be in "normal adjustment". The angular magnification of a microscope in normal adjustment is defined as follows:

$$Angular\ magnification\ (m) = \frac{\begin{array}{c}Angle\ subtended\ at\ the\ eye\ by\ the\ image\\ at\ the\ near\ point\end{array}}{\begin{array}{c}Angle\ subtended\ at\ the\ eye\ by\ the\ object\\ when\ placed\ at\ the\ near\ point\end{array}}$$

It should be noted that the image at the near point in the normal adjustment gives the maximum magnification in a microscope.

The simple microscope

The simple microscope, also known as the magnifying lens, consists of a single convex lens of short focal length. The object to be viewed is placed in front of the lens within its principal focus to form a virtual, magnified, upright image. When the object is adjusted to form this image at the near point, the microscope is in normal adjustment.

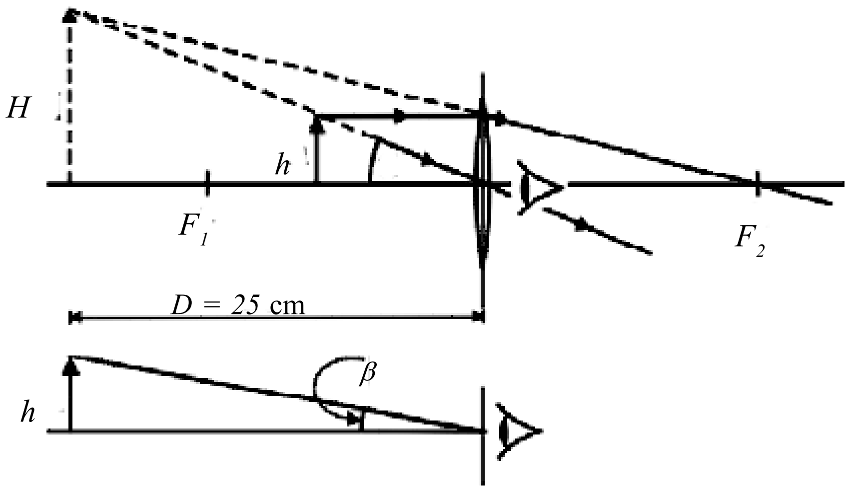


Figure 11.2

Angular magnification $(m) = \frac{\alpha}{\beta} = \frac{\tan\alpha}{\tan\beta} = \frac{H/D}{h/D} = \frac{H}{h}$

Thus for the simple microscope the angular magnification is the same as linear magnification

$\frac{H}{h} = \frac{Height\ of\ image}{Height\ of\ object}$

Geometrically it can be proved that, $\frac{H}{h} = \frac{v}{u}$

Hence for the simple microscope in normal adjustment $m = \frac{v}{u}$

Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ and applying the sign convention, $\frac{1}{v}-\frac{1}{u}=\frac{1}{-f}$

By multiplying the above equation by v $1-\frac{v}{u}=\frac{-v}{f}$

$$1-m=\frac{-D}{f}$$

$$m=1+\left|\frac{D}{f}\right|$$

The compound microscope

The compound microscope uses two lenses to magnify the image twice to obtain a higher magnification. The lens closer to the object and known as the "objective" is a converging lens of very short focal length. The lens closer to the eye and known as the "eye piece" is also a converging lens of short focal length but longer than that of the objective.

The object to be viewed is placed a little beyond the focus of the objective lens, to form a real, inverted and magnified image I_1. The eye-piece is then placed so that the image I_1 falls within its principal focus. This image I_1 will then act as the object to the eye-piece and would cause the formation of the final image which is further magnified, inverted and virtual. When the eye-piece is adjusted to have the final image at the near point, the microscope is in normal adjustment.

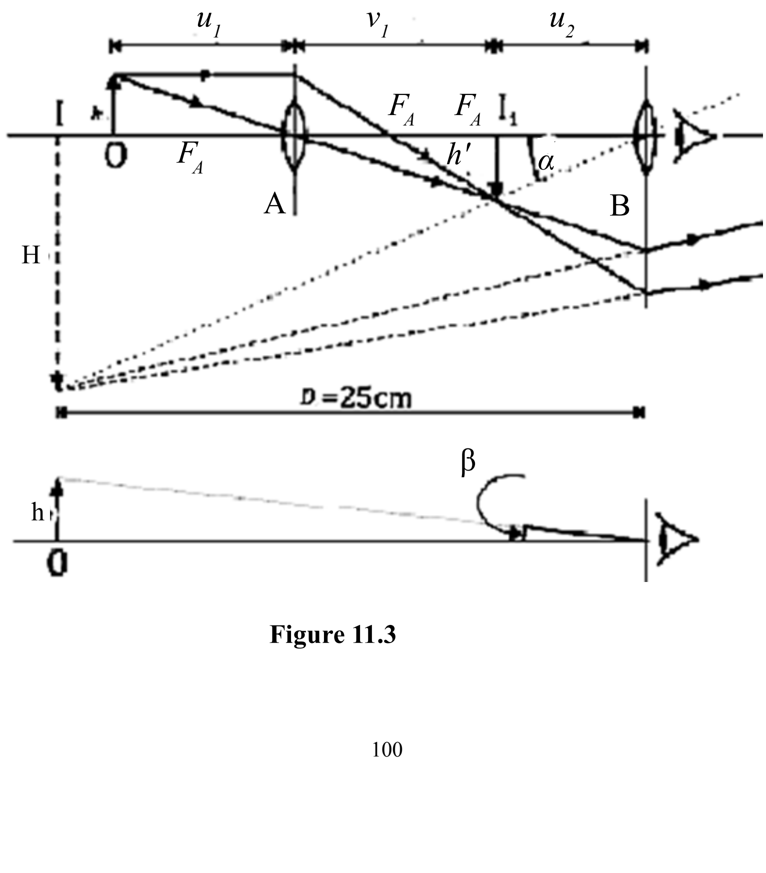


Figure 11.3

Angular magnification $(m) = \frac{\alpha}{\beta} = \frac{\tan\alpha}{\tan\beta} = \frac{\frac{H}{D}}{\frac{h}{D}} = \frac{H}{h} = \frac{H}{h_1}.\frac{h_1}{h}$

$$m = \frac{D}{u_2} \cdot \frac{v_1}{u_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 11.1$$

Applying $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ with sign convention for,

the eye piece B : $\frac{1}{D} - \frac{1}{u_2} = \frac{1}{-f_B}$

By multiplying the above equation by D

$$1 - \frac{D}{u_2} = -\frac{D}{f_B} \Rightarrow \frac{D}{u_2} = 1 + \left|\frac{D}{f_B}\right|$$

the Objective A: $\frac{1}{-v_1} - \frac{1}{u_1} = \frac{1}{-f_A}$

$$\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f_A}$$

By multiplying the above equation by v_1

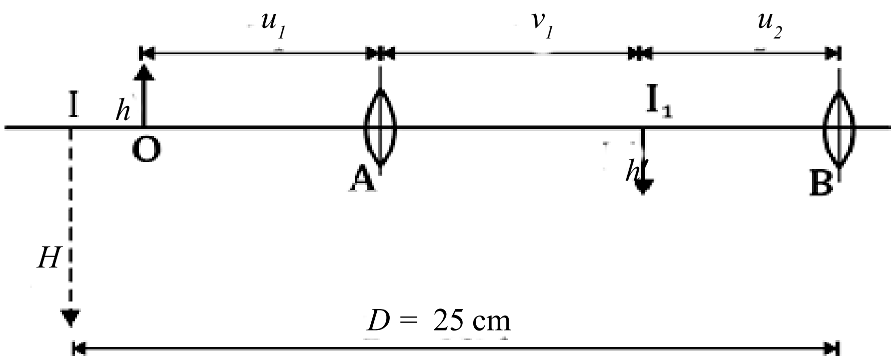
$$1 + \frac{v_1}{u_1} = \frac{v_1}{f_A} \quad \Rightarrow \quad \frac{v_1}{u_1} = \left|\frac{v_1}{f_A}\right| - 1$$

Substituting for $\frac{D}{u_2}$ and $\frac{v_1}{u_1}$ in equation .11.1

$$m = \left\{1 + \left|\frac{D}{f_B}\right|\right\} - \left\{\left|\frac{v_1}{f_A}\right| - 1\right\}$$

Worked Example

A compound microscope has an objective lens of focal length 2 cm and an eye-piece of focal length 5 cm. An object is placed 2.5 cm from the objective and the final image is formed 25 cm from the eye. What is the magnification produced by the microscope and the separation of the lenses?



As proved earlier, $m = \frac{D}{u_2} . \frac{v_1}{u_1}$

For the eye-piece using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{25} - \frac{1}{u_2} = \frac{1}{-5} \quad \Rightarrow \quad \frac{1}{u_2} = \frac{1}{25} + \frac{1}{5} = \frac{6}{25} \Rightarrow u_2 = \frac{25}{6}$$

For the objective using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v_1} - \frac{1}{2.5} = \frac{1}{-2} \quad \Rightarrow \quad \frac{1}{v_1} = \frac{1}{2.5} - \frac{1}{2} = \frac{-5}{50} = -\frac{1}{10}$$

$$v_1 = -10 \, ; \ \therefore m_B = \frac{D}{u_2} = \frac{25}{25/6} = 6 \ ....(1)$$

$$\therefore m_A = \frac{v_1}{u_1} = \frac{10}{2.5} = 4 \ ....(2)$$

From (1) × (2) Angular magnification $m = m_A \times m_B = 4 \text{ x } 6 = 24$

Separation between the two lenses $= v_1 + u_2 = 10 + \frac{25}{6} = \frac{85}{6} = 14\frac{1}{6}$ cm

Telescopes

Telescopes are optical instruments used to view clear magnified images of distant objects such as astronomical objects.

When using telescopes it is customary for the final image to be formed at infinity. When the final image is at infinity, the telescope is said to be in "normal adjustment". The angular magnification or the magnifying power of a telescope in normal adjustment is defined as follows.

$$\text{Angular magnification } (m) = \frac{\text{Angle subtended at the eye by the image at infinity}}{\text{Angle subtended at the eye by the object at infinity}}$$

An advantage of the normal adjustment is that the eye is most relaxed when viewing the image when it is formed at infinity.

The astronomical telescope

The astronomical telescope consist of an objective lens, which is a convex lens of long focal length, while the eye-piece too is a convex lens but of short focal length.

The objective forms a real, inverted and magnified image of a distant object at its secondary principal focus. The eye-piece is then placed so that this image falls within its principal focus. This image would then be an object to the eye-piece to form the final image which would be virtual, inverted and magnified. When the eye-piece is adjusted to form the final image at infinity, the telescope is in normal adjustment. This would take place when the principal focus of the eye-piece coincides with the first image I_1 formed by the objective.

Normal adjustment

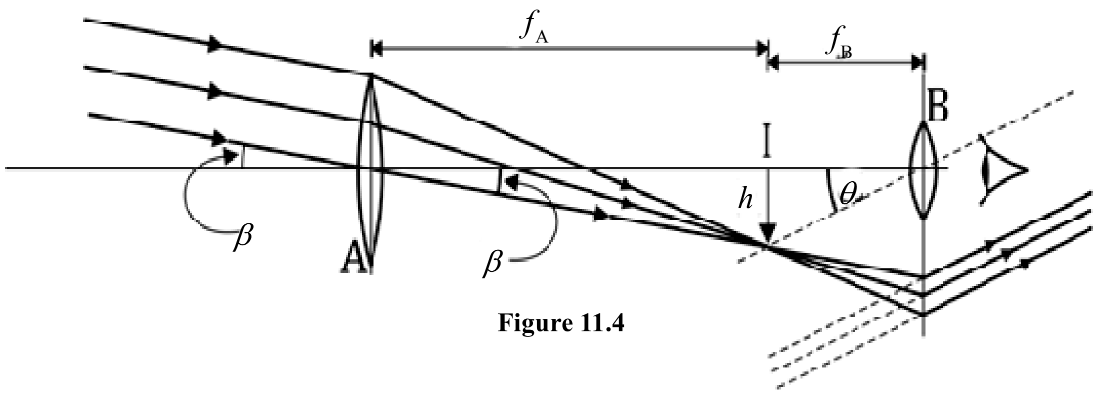


Figure 11.4

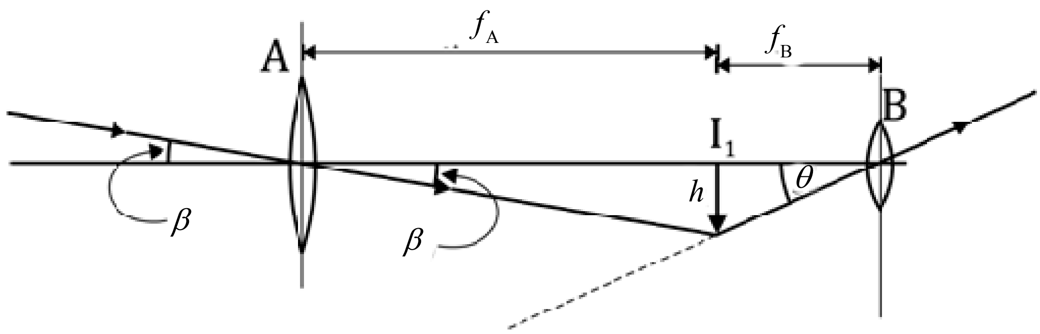
$$\text{Angular magnification}, (m) = \frac{\alpha}{\beta} = \frac{\tan\alpha}{\tan\beta} = \frac{h/f_B}{h/f_A} = \left|\frac{f_A}{f_B}\right|$$

$$\text{Distance between the two lenses} = |f_A| + |f_B|$$

Worked Example

An astronomical telescope in normal adjustment consists of two convex lenses placed 100 cm apart and the magnification produced is 24. Find the focal lengths of the objective lens and the eye-piece.
If the eye-piece has to be pulled out a distance of 4 cm to focus on a near object how far away is this object from the objective? What is the magnification in the new adjustment?

Solution



$$m = \frac{f_A}{f_B} = 24 \quad ....(1)$$

Also $f_A + f_B = 100$ ....(2)

Solving equations (1) and (2),

$f_A = 96$ cm $\qquad f_B = 4$ cm

Assuming that the final image is at infinity for the near object too, the 4 cm pull of the eye-piece means that the first image I_1 is formed at 100 cm from the objective.

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for the objective,

$$\frac{1}{-100} - \frac{1}{u} = \frac{1}{-96}$$

$$\frac{1}{u} = \frac{1}{96} - \frac{1}{100} = \frac{100-96}{9600} = \frac{4}{9600}$$

Object distance $u = 2400$ cm

New magnification $= \frac{100}{4} = 25$

The eye ring

When viewing images formed by microscopes and telescopes, an important factor is the position of the eye which would provide the best view as far as the brightness of the image is concerned. For this feature to be included, let us consider the image formed by an optical instrument consisting of two lenses – the objective and the eye-piece.

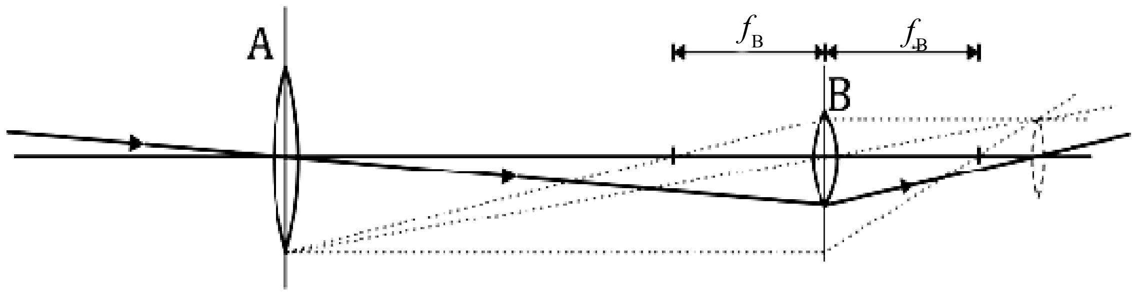


Figure 11.5

The ray passing through the optical centre of the objective O, after refraction through the eye-piece E, intercepts the axis of the instrument at a certain point where the eye-piece forms an image of the objective. This is known as the "eye-ring" and is considered as the best position to place the eye to view the final image, since it is at the eye ring that all the light passing through the objective and then through the eye-piece will be most concentrated to provide maximum brightness.

Calculation of the position of the eye ring can be done by applying the lens formula to the eye-piece and taking the objective as the object.

There can be occasions when microscopes and telescopes are used in situations, where the instruments are not in normal adjustment.

For example

(1) Compound microscope adjusted for the final image to be at infinity.

(2) Astronomical telescope adjusted for the final image to be at the near point.

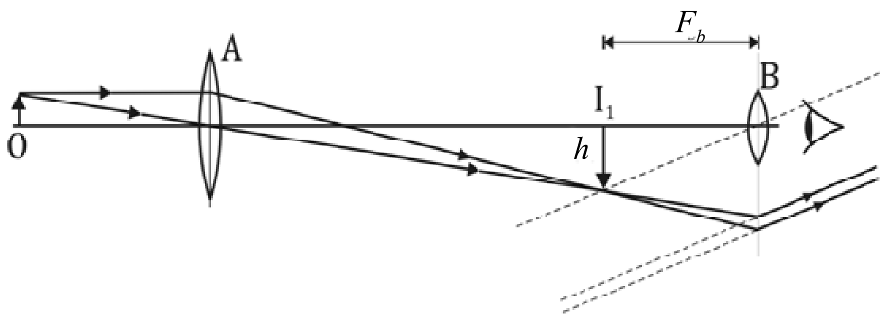


Figure 11.6

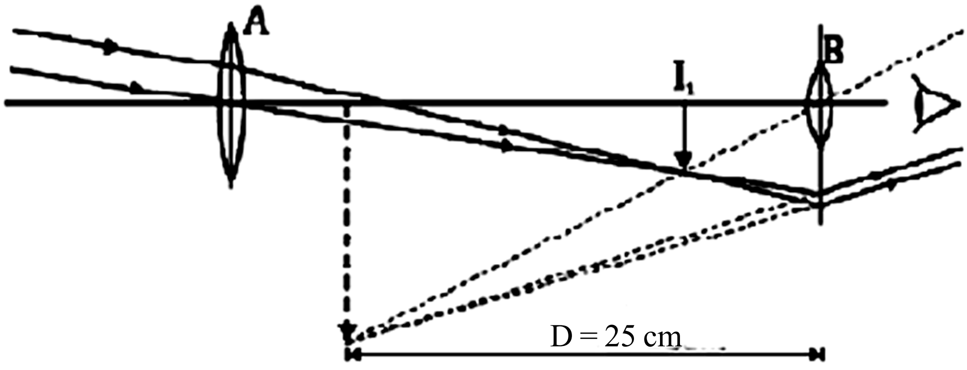


Figure 11.7

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