

General Certificate of Education (Advanced Level)

BUSINESS STATISTICS

Grade 13

**Teacher's Instructional Manual
(Implemented from year 2010)**



**Department of Business Studies
Faculty of Science & Technology
National Institute of Education**

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STATISTICS

Teacher's Instructional Manual

Grade 13 – 2010

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Introduction

Curriculum developers of the NIE were able to introduce Competency Based Learning and Teaching curricula for grades 6 and 10 in 2007 and were also able to extend it to 7, 8 and 11 progressively every year and even to GCE (A/L) classes in 2009. In the same manner, syllabi and Teacher's Instruction Manuals for grades 12 and 13 for different subjects with competencies and competency levels that should be developed in students are presented descriptively. Information given on each subject will immensely help the teachers to prepare for the Learning – Teaching situations.

I would like to mention that curriculum developers have followed a different approach when preparing Teacher's Instruction Manuals for Advanced Level subjects when compared to the approaches they followed in preparing Junior Secondary and Senior Secondary curricula (Grades 10, 11).

In grades 6, 7, 8, 9, 10 and 11 teachers were oriented to a given format as to how they should handle the subject matter in the Learning – Teaching process, but in designing AL syllabi and Teacher's Instruction Manuals, freedom is given to the teachers to work as they wish.

At this level we expect teachers to use a suitable learning method from the suggested learning methods given in the Teacher's Instruction Manuals to develop competencies and competency levels relevant to each lesson or lesson unit.

Whatever the learning approach the teacher uses, it should be done effectively and satisfactorily to realize the expected competencies and competency levels.

I would like to note that the decision to give this freedom is taken, considering the importance of GCE (A/L) examinations and the sensitivity of other stakeholders who are in the education system to the Advanced Level examination. I hope that this Teacher's Instruction Manual would be of great help to teachers.

I hope the information, methods and instructions given in this Teacher's Instructional Manual will provide proper guidance to teachers to awaken the minds of our students.

Dr. Upali Sedere
Director General
National Institute of Education

Foreword

Action taken over long years of the past to retain the known and learn the predetermined has made us little able today to construct even what is. The first curriculum reform of the new millennium on secondary education that comes to being with a drastic change in the learning-teaching process at school level attempts to overcome this inability while bringing about a set of worthy citizens for the country who are capable of revising the known, exploring the undetermined and constructing what might be.

If you are a teacher teaching this subject or any other subject in grades 6 to 11, it will not be difficult for you to align yourself with the new learning-teaching approaches that are recommended in a considerable way for the GCE (A/L) as well. This reform calls the teacher to identify competency levels under each competency and plan activities to achieve them. The teachers entering the new role of transformation should understand that the procedures which emphasize the teacher in the learning-teaching process are of limited use for the present and that it is more meaningful for the children to learn co-operatively sharing their experiences. This situation, however, requires the teachers to provide a new direction for their teaching by selecting new learning-teaching methods that emphasize the student over the teacher.

If you study the Teachers' Instructional Guides (TIGs) prepared by the National Institute of Education for Mathematics, Science, Health and Physical Education, Technology and Commerce subjects of grades 6 to 11, you certainly will be able to acquire a good understanding on the student-centred, competency based and activity-oriented approaches we have recommended for learning and teaching. The activities presented in these Guides attempt to bring learning, teaching assessment and evaluation on to the same platform and to help you to adopt co-operative learning techniques on the basis of the 5E Model.

Considering the need to establish an innovative teaching force we have selected just a few activities from the relevant activity continuum incorporated in the TIGs. Yet we have given you a vast freedom to plan your own activities to suit the subject and the class requirements by studying the exemplar activities in the Guides and improving your understanding on the principles underlying the reform. The activities incorporated in the TIG, provide you with four types of information. At the beginning of each activity you come across the final outcome that the children are expected to achieve through each activity. This learning outcome named as 'Competency' is broad and long-term. The competency level stated next highlights one out of the number of abilities that the children have to develop to realize the competency.

The above explanation shows us that the competency levels are more specific and of a shorter duration when compared to the competency. The next section of the Guide presents a list of behaviours that the teacher has to observe at the end of each activity. To facilitate the task of both the teacher and the students, an attempt has been made to limit the number of such behaviours to five. These behaviours referred to as learning outcomes are more specific than the competency level. They include three abilities derived from the subject and two others derived from the learning teaching process. Out of the three subject abilities listed in an order of difficulty, the teacher has to direct the children to realize at least the first two through the exploration. The next section of the activity presents what the teacher should do to engage the children for the exploration. Although the implementation of each and every activity starts with this step of engagement, the teachers

should not forget that activity planning should begin with the exploration which is the second 'E' of the 5E Model.

Instructions for the group exploration from the next section of the exemplar activities the teacher plans these instructions in such a way to allow different groups studying different facets of the same problem to reach the expected ends through a variety of learning-teaching methods. For this, further the teacher can select either Inquiry-based Learning carried out through a series of questions or Experiential Learning where children learn by doing. It is the responsibility of the GCE (A/L) teacher to use the knowledge that the children acquire by any of the above methods to solve problems that are specific to the subject or that runs across a number of subjects of the curriculum is meaningful to plan such problem-based learning-teaching methods on the basis of real-life situations. For this you can select dilemmas, hypothetical situations, analogies or primary sources. Some techniques that can be used for the explorations are reading, information management, reflection, observation, discussion, formulation and testing of hypotheses, testing predictions, preparing questions and answers, simulation, problem solving and aesthetic activities such as drawing or composing. There is room here even for memorization although it is considered as a form of mechanical learning.

The students explore in small groups. Instead of depending on the knowledge available to the teacher, they attempt to construct their own knowledge and meaning with the support of the teacher. Moreover, they interact with others in the group to learn from others and also to improve the quality of their exploration findings. All this works successfully only if the teacher is capable of providing the students with the reading material and the other inputs they are in need of. The teacher also has to support student learning throughout the learning process by moving from one group to another. Although it is the discovery that is prominent in this type of learning you have to recognize this as a guided discovery rather than a free discovery. There is no doubt that students learning likewise with instructional scaffolding both by the teacher and the peers acquire a whole lot of worthwhile experiences that they find useful later in life.

Explanation follows the second stage of exploration. The small groups get ready to make innovative, team presentations on their findings. The special feature here is that the children have selected novel methods for their presentations. The responsibility for the presentation is also shared by all members of the group. In the next step of elaboration the children get the opportunity to clarify the unclear, correct the incorrect and fill any gaps that are left. They also can go beyond the known to present new ideas. All activities end with a brief lecture made by the teacher. This stage allows the teacher to go back to the transmission role. The teacher also has to deliver this lecture covering all the important points that the syllabus has prescribed for the relevant competency level. Step 3 of each Activity Plan guides the teachers in this compulsory final elaboration.

To overcome many problems that are associated with the general system of education today, the National Institute of Education has taken steps to move the teachers to the new transformation role recommended for them. This role that starts with a transaction gets extended to a lengthy exploration, a series of student explorations and elaborations and a summative transmission by the teacher. The students involve themselves in the exploration using reading material and other quality inputs provided to them by the teacher.

The students attend school daily to learn joyfully. They achieve a number of competencies that they need to be successful in life and the world of work. They prepare themselves for nation building by developing thinking skills, social skills and personal skills. For the success of all this, an examination system that inquires into the ability of students to face real challenges of life is very much needed in place of an examination system that focuses on the knowledge acquired by children through memorizing answers to model questions.

A number of activities have already begun at the national level to protect the real nature of school-based assessments. The written tests have been minimised to gain recognition for school-based assessments. Compulsory question has been incorporated in the term tests along with a scheme of authentic evaluation to ensure real outcomes of learning. It is the co-ordinated responsibility of all citizens of the country to open up doors for a new Sri Lanka by thriving for the success of this new programme on the basis of sound instructional leadership and quality assurance by the management.

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**LEARNING OUTCOMES
AND
MODEL ACTIVITIES**

Competency 6.0 : Makes Business Decisions

Competency Level 6.1 : Uses Statistical Inference in order to make decisions in Business field.

Number of periods : 04

Learning Outcomes :

- Highlights the differences between Parameters and Statistics.
- Differentiates the Census and the Sample Surveys.
- Names the general measures of summary and builds up events to explain them.
- Explains the way on which the conclusions are made based on sample data.
- Demonstrates the ability of performance efficiently.
- Uses methods to arrive at quick and optimal decisions.

Learning - Teaching Process :

Approach

- Call upon two volunteer students before the class and make them present the following dialogue.

Mother : Shanthi, I'm sick today. Can you prepare the lunch please.

Shanthi : Of course mother .. But I don't know how to check whether the rice is boiled enough.

Mother : When the water level is evaporated take out small sample of the rice and thresh it, then you can decide on the entire pot of rice whether it is boiled or not.

- Lead the discussion highlighting the following facts.
 - When the population is large the sample study is very important.
 - The Behavior of population can be studied using sample data.
 - Accurate sampling methods should be followed for the success of the conclusions about the population.

Proposed Instructions for learning :

- Provide the following pairs of topics among the groups.
- Census and Parameters
- Sample Surveys and Statistics
- Let them explain the pair of topics using an example related to the out put of a manufacturing firm. (Instruct the students to build up events related to the output using assumed data.)
- Let them compute the summery measures with regard the situation they have received.
- Lead them to explain the qualities of those summery measures.
- Enquire how to use those measures for Statistical Inference.
- Enquire students about the population from which the sample is selected and the population at which the conclusions are targeted in the example constructed by students.
- Let them to review about the uses and problems related to each opportunity given to the students.
- Guide them to propose appropriate actions to overcome those problems.

Guidelines to understand the subject matters:

- Lead a discussion highlighting the following facts.
 - The process of collecting data from all the units of the populations known as the census.
 - The process of studying a representative part taken from the population is known as a Sample Survey.
 - The population for which the conclusions should be made through sample surveys is the target population.
- The population actually covered by the sample is the sampled population
- Measures on which the distribution of a population is based on are known as ‘parameters’

Ex:

- Population Mean μ
- Population Variance σ^2
- Population Proportion π
- Population Median M_d

- Parameters are always constant.
- A statistic is a function of sample elements.
- A statistic is a random variable.

Ex:

- Sample mean \bar{X}
- Sample Proportion P

- Hence there are probability distributions for those statistics.
- The probability distribution of a statistic is known as a sampling distribution.
- Population parameters are estimated based on sampling distribution.
- Making conclusions about the parameters using the statistics computed in the sample studies is the statistical inference.
- Statistical inference consists of two main sections as follows
 - Statistical estimations
 - Statistical hypothesis testing

Competency Level 6.2 : Uses Random sampling methods suitable for the situations.

Number of periods : 16

Learning Outcomes :

- Names possible random sampling methods applicable for a particular situation.
- Examines the relative merits and demerits of those sampling techniques.
- Selects random samples with and without a sampling frame.
- Demonstrates the need of minimizing the business risks.
- Selects the optimal sampling techniques appropriate to the situation.

Learning - Teaching Process :

Approach

- Select students from the class in following ways.
 - First five students seated in front row.
 - Any five of the students by drawing lotteries.
- **Lead a discussion highlighting the following facts.**
- Five students were selected in random by drawing lotteries.
- A sampling frame was used for that task.
- Any probability base was not used to select the five students seated in front row.
- Population is well represented by random samples.
- The results derived from such sample studies are used for statistical inferences.
- Selection of Samples without probability base may lead for bias.

Proposed Instructions for learning:

- **Distribute the following situations among the groups of students.**
- Selecting a sample of 50 machine operators from a garment factory in which 500 are working.
- Selecting a sample of 50 items from a firm manufacturing the commodities toothpaste, soap, shampoo, baby cream and baby powder for the purpose of awarding quality certificates by examining their quality.
- Selecting a sample of 50 commuters for a study launching to examine the problems faced by those who travel in private coaches running in Kandy – Colombo highway per day.
- Selecting a sample of 50 vouchers from 50 sets of paying vouchers of 10 in each in accounts branch of a particular business firm .
- Guide the students to propose the best sampling method to select a representative sample fit for the situation received by each group.
- Explain how to select the relevant sample using that technique.
- Give the guidance to select the sample using hypothetical data.
- Give the guidance to explain the eligibility of the sampling technique selected by your group for the situation that you have received.
- Explain the possible problems in using that sampling technique.
- Explain some other instances for which this sampling technique may be applicable.
- Prepare for a collective and constructive presentation with your findings before the entire class.

Guidelines to understand the subject matters:

- **Lead the discussion by highlighting the following facts.**
- Selection of a samples so that every sample unit has known probability to be included in the sample is probability sampling.
- The sampling method of selection of samples giving equal probability for every population unit to be included in the sample is simple random sampling.
- This probability is $\frac{1}{N}$.
- The results of obtained in analyzing the data collected from this sample can be used in statistical inference.

- When the population consists of several strata, the selection of a sample from each strata is named as Stratified random sampling.
 - In the Stratified random sampling, Inferences can be made for the entire population as well as for the sub populations.
- Field work and supervision are convenient in stratified random sampling than the simple random sampling.
 - Sampling frame is necessary for both the simple random sampling and stratified random sampling.
- The following methods are used to decide the sample size and to allocate the sample between stratus in the stratified sampling.
 - Equal allocation
 - Proportional allocation
 - Optimum allocation
- Cluster sampling is the method of selecting a cluster or several clusters randomly as the sample from the population which consists of elements of clusters.
- Under the following methods cluster sampling can be done
 - Single stage cluster sampling
 - Two stage cluster sampling
 - Multi stage cluster sampling
- When the primary sampling unit is geographical area, such sampling is called area sampling and it is a special situation of the cluster sampling.
- Cluster sampling is mostly appropriate when the sampling frame dose not exist.
- Stratified sampling is mostly appropriate when the variation within the cluster is small and the variation between the clusters is large.
- Cluster sampling is mostly appropriate when the variation within the cluster is large and the variation between the clusters is small.
- In systematic sampling the population is first divided in to n groups of size k such that $k = \frac{n}{N}$. Then a unit is selected randomly from the first group of size k and there after every k^{th} unit is selected for the sample of size n .
- Using accurate methods of sampling according to the situation well represented samples can be selected and this lead to a successful study.

Competency Level 6.3 : Selects appropriate samples following non random sampling methods.

Number of periods : 10

Learning Outcomes :

- States the name of non random sampling methods
- Gives the situations where such samples are more appropriate.
- Explains the advantages and disadvantages of non random sampling methods comparing with random methods
- Demonstrates the preparedness to make quick decisions accurately
- Takes appropriate actions to minimize error by studying them..

Learning - Teaching Process:

Approach

- Presents the following statements to the class
“A television channel wants to make a description about the prices of goods during last week at an economic centre”
- Ask the students about the way of selecting samples collecting data for this programme.
- **Lead the discussion to highlight the following facts.**
 - The intention of the television channel is to inform the public about the current situation of the price of the goods as soon as possible.
 - Hence, it’s necessary to collect correct information and make the public aware of the prevailing situation quickly.
 - There are situations where simple sampling methods can be applied to obtain quick results.
 - Practically it is difficult to use random sampling methods.

Proposed Instructions for learning:

- Assign the following cases to group of students.
- It is required to select an A/L student to deliver a speech at the morning assembly tomorrow in the school.
- It is required to select a sample representing children/adults, male/female, employed/unemployed for a market survey for a brand of toothpaste.
- The human resource manager of a business firm has to make a report on the steps taken for the welfare of employees after obtaining the opinion of employees.
- Give the instruction to the students to suggest a suitable sampling method for each case.
- Direct the students to describe how to select the relevant sample step by step.
- Direct the students to suggest another situation where the same sampling method can be applied to select a sample.
- Give the opportunity for the students to state merits and demerits of sampling methods suggested by them.
- Instruct the students to explain the errors which may occur in the process of sampling and in the process of analyzing sampling data.
- Prepare for a collective and constructive presentation with your findings before the entire class.

Guidelines to understand the subject matters:

- Three types of nonrandom sampling methods can be identified.
- Convenient sampling.
- Judgment sampling.
- Quota sampling.
- The method of drawing a sample taking the units of the population which can be accessed easily is known as convenient sampling.
- This method is more suitable in a simple investigation where quick results are required.
- The method of drawing a sample under the guidance of a specialist in the relevant field is known as judgment sampling.

- Judgment sampling is appropriate in following situations.
 - When the cost of sampling is high.
 - When sample size is very small.
 - When the population is much heterogeneous.
 - Drawing a nonrandom sample representing the members of each sub group of the population is known as the Quota sampling.
 - Sub groups are known as Quota controls.
 - Quota controls consists of the following two categories.
 - Independent Quota controls.
 - Inter-related Quota controls.
 - Sub group such as sex, age are independent Quota controls where as employment, (profession) income group and education standard are known as Inter-related Quota controls.
-
- The interviewer selects the sample in the field according to the quota control scheme assigned to him/her.
 - A well representative sample may be drawn based on the knowledge, training and experiences of the interviewer.
 - There are two types of possible errors in sampling.
 - Sampling errors.
 - Non sampling errors.
-
- The standard error or standard deviation of the sampling distribution of a statistics used to estimate the population parameters is known as sampling error.
 - Sampling error can be minimized by using large sample size and selecting a well represented sample.
 - The errors which may occur in recording, copying or computing in sample investigation are known as non sampling errors.
 - Such errors occur even in censuses.

Competency Level 6.4 (a): Examines the relationship between statistics and parameters for making business decisions

Number of periods : 04

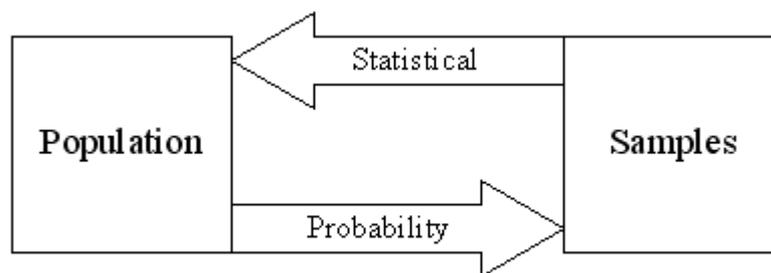
Learning Outcomes :

- Selects all possible samples from a hypothetical finite population
- Computes sample Statistics.
- Explains the relationships between Population Mean and Sample Mean
- Uses the concept of probability in Decision making
- Makes decisions identifying relationships

Learning - Teaching Process:

Approach

- Presents the following model to the students



- **Lead the discussion by highlighting the following facts.**
- The probability is used in making decisions about the population based on the sample data obtain from that population.
- The sample size and the precision of an estimator are required for making accurate conclusions

.Proposed Instructions for learning:

- Values of a characteristics of a hypothetical population of size 5 are given below.
2, 6, 4, 3, 7
- Give instructions to groups of students to select all possible samples without replacement from this population for the sample size given below.
- Sample size of 2
- Sample size of 3
- Give instruction to group of students to establish relationships between parameters and statistics by performing the following calculations.
- Find the population mean and variance.
- Construct the probability distribution table of \bar{X} .
- Denote the sample means by \bar{x} and the probability of each \bar{x} by $P(\bar{x})$.
- Calculate mean and variance of the probability distribution \bar{X} .
- Show that the mean of sample means is the same as the population mean.
- Verify that the variance of the sample mean is given by $\frac{\sigma^2}{n} \left(\frac{N-n}{N-1} \right)$
- Prepare for a collective and constructive presentation with your findings before the entire class.

Guidelines to understand the subject matters:

- The number of samples of size n that can be taken from a population of size N is ${}^N C_n$
- The sample mean \bar{X} of the all possible samples of size n from a population is a random variable.
- The probability distribution of the random variable \bar{X} is called the sampling distribution of \bar{X} .
- The sampling distribution of the sample mean, \bar{X} has the following characteristics

- $\mu_{\bar{X}} = \mu \quad [E(\bar{X}) = \mu]$
- When sampling from an infinite population the variance of the sample mean \bar{X} is $V(\bar{X}) = \frac{\sigma^2}{n}$.
- If the population is finite the variance of the sample \bar{X} is $V(\bar{X}) = \frac{\sigma^2}{n} \left(\frac{N-n}{N-1} \right)$
- The term $\frac{N-n}{N-1}$ is called finite population correction.

Competency Level 6.4(b) : Construct sampling distribution of the sample mean for making business decisions

Number of periods : 08

Learning Outcomes :

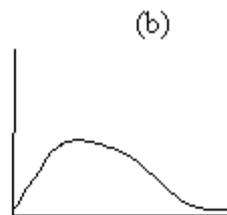
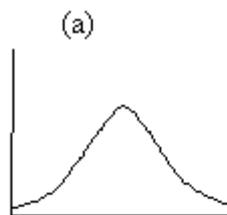
- Selects the conditions that required to identify the nature of a sampling distribution
- States the Central Limit Theorem
- Calculates the probability that a parameter lies in a given interval of the population based on the sampling distribution.
- Demonstrates the readiness to work within the existing constraints

Learning - Teaching Process:

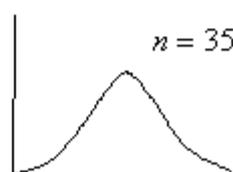
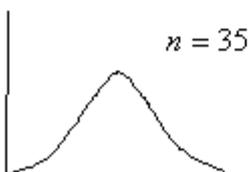
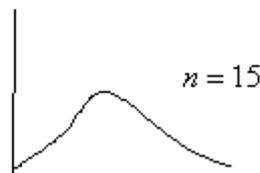
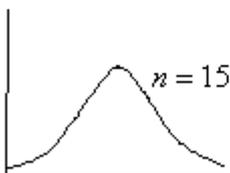
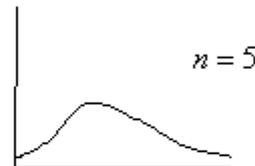
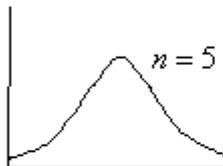
Approach

- Present the following models to the students

Population Distributions



The distributions of the sample mean \bar{X} for different sample sizes



- Lead a discussion highlighting the following facts.
- The first two diagrams (a) and (b) shows two different population distributions.
- The distribution of (a) is a normal distribution while the distribution of (b) is non-normal distribution.
- The other diagrams show how the distribution of \bar{X} approaches the symmetrical shape as the sample size increases.

Proposed Instructions for learning:

- Assign the following cases to student groups to construct sampling distributions.
- It is required to issue a certificate on the maximum weight that can be beared by a certain type of steel wire produced by a steel production firm. The maximum weight that can be beared by wires has a normal distribution. Consider the following cases also.
- The variance of the maximum weight is 250 kg .
- The variance of the maximum weight is un known .
- The data collected for a sample of size 36 has the mean weight of 500 kg , with the variance 145 kg. It is required to calculate the probability that weight which can be beared by the wire is between 465 kg and 520 kg.
- Further it is required to decide the sample size to maintain the maximum weight of the wire stock at a level of at least 10% of the stock for the mean to be 550 kg without changing the variance of the weight.
- It is required to make decisions about the salaries of technical officers in a factory, It is known that salaries of employees has a normal distribution. For the data collected for a sample of size 16 employees, the mean wages and standard deviation were Rs. 40,000/= and Rs. 5000/= respectively. It is required to decide the minimum wage of 10% of the employees who received highest salaries in the sample and the maximum wages of 10 % of the employees who received lowest salaries.
- It is required to make the decision about the number of persons who enter a working place in a working day. In a 15 minutes interval the mean rate of arrival of persons to this working place is 2 and the standard deviation is $\sqrt{2}$. For 100 intervals of 15 minutes the number of arrivals were was recorded. It is required to consider the probability that the mean of the number of arrivals in the interval considered is less than 1.85.

- It is required to find the way of changing the standard deviation in such a way that the probability of the mean of number of arrivals is less than 20% without changing the sample size and the mean.
- Give instructions to student groups to provide answers to following questions.
- State the distribution relevant to the given distribution.
- Comment on the population variance/Standard Deviation.
- State the sampling distribution of \bar{X} relevant to the case being considered.
- Write down the conditions if any you have to use to state the sampling distribution of \bar{X} .
- State the sampling distribution of \bar{X} relevant to each case.
- Find the probability that should be computed relevant to the case you received using the sampling distribution.
- Perform the required calculations making changes stated in each given case.
- Prepare for a collective and constructive presentation with your findings before the entire class.

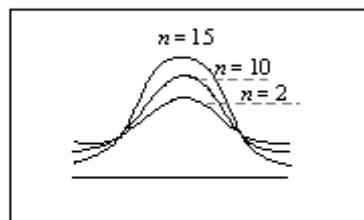
Guidelines to understand the subject matters:

- The following three questions should be raised in deciding the sampling distribution of the sample mean \bar{X} .
- Whether the population is normal or not.
- Whether the sample size is large or small.
- Whether the population variance is known or not known.
- The mean \bar{X} of sample size n from a normal distribution with known variance can be explained by a normal distribution with mean μ and variance $\frac{\sigma^2}{n}$.

- This can be stated as
$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

- When a large random sample of size n is drawn from normal distribution with unknown variance the sample mean \bar{X} can be explained approximately by a normal distribution with mean μ and variance $\frac{\sigma^2}{n}$.

- If a sample size n drawn from a normal distribution with known variance is small the sample mean \bar{X} can be explained by a normal distribution with mean μ and variance $\frac{\sigma^2}{n}$.
- This can be stated as $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$
- If a sample size drawn from a normal distribution with unknown variance is small $\frac{\bar{X} - \mu}{s/\sqrt{n}}$ has a t -distribution with $n - 1$ degrees of freedom.
- If the sample size is sufficiently large ($n \geq 30$) the sample mean \bar{X} has a normal distribution approximately as stated by the Central Limit Theorem.
- The Student's t distribution depends on the degrees of freedom $n - 1$.
- For small samples spread of the t distribution is larger than the spread of the standard normal distribution.
- As sample size increases, the t distribution approaches the standard normal distribution.
- The following figure shows the above fact.



Competency Level 6.5 : Constructs sampling distribution for difference of two sample means

Number of periods : 10

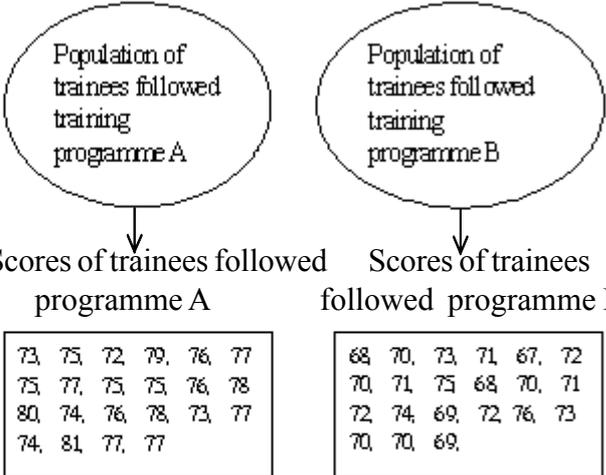
Learning Outcomes :

- Names mean and variance of sampling distribution for differences of two sample means.
- Constructs sampling distributions for difference of two samples means.
- Calculates the probabilities for the difference of means of random samples taken from the two normal populations
- Demonstrates the preparedness to use the sampling distributions for difference of sample means for the business decisions.
- Takes decisions relevant to difference of means of two variables (Two populations) calculating risk under uncertainty.

Learning - Teaching Process:

Approach

- Guide students to study the plan that can be used to take the following decisions

| | |
|---|--|
| <p>Step 1</p> <p>Comparison of two management training programmes.</p> <p>Data: Test scores of each sample</p> |  <p>Population of trainees followed training programme A</p> <p>Population of trainees followed training programme B</p> <p>Scores of trainees followed programme A</p> <p>Scores of trainees followed programme B</p> |
| <p>Step 2 Data Organization and Analysis(Descriptive Statistics)</p> |  <p>$\bar{X} = 76$</p> <p>$\bar{X} = 71$</p> |
| <p>Step 3 Taking of Decisions(Inferential Statistics)</p> | <ul style="list-style-type: none"> • The difference between above two sample means is 5. • If any other samples are taken from two populations, another value may be obtained for this difference. |

- Step 1 Experiment: Comparison of two management training programmes. Data: Test scores of each sample Scores of trainees followed
- Step 2 Data Organization and Analysis(Descriptive Statistics) $\bar{X} = 76$ $\bar{Y} = 71$
- Step 3 Taking Decisions(Inferential Statistics)
- The difference between above two sample means is 5.
- If any other samples are taken from two populations, another value may be obtained for this difference.
- Lead a discussion highlighting the following facts.
- It is required to test whether there is a difference between means of two proportions for some studies.
- Decisions can be made using random samples instead of studying the population.
- The difference between two sample means is a random variable.
- A sampling distribution exists for difference between two sample means.

Proposed Instructions for learning:

- Provide the following two cases to the two groups of students.
- There are two companies A and B which produce light bulbs. The life time of the bulbs produced by company A is distributed normally with mean 1400 hrs and standard deviation 200 hrs. The life time of the bulbs produced by company B is also distributed normally with mean 1200 hrs and Standard deviation 100 hrs. It is required to test whether there is a difference between life times of bulbs produced by two companies.
- It is required to calculate the probability that mean lifetime of bulb A is greater than the mean life time of bulb B is
 - at least 160 hrs more.
 - less than or equal to 100 hrs.
- It is also required to decide the size of the sample that ensures the probability of having the difference is less than 100 hrs at the 0.05 level.
- The following information is given by two companies which produce two types of tires

A and B.

$$\begin{array}{ll} \mu_x = 5000km & \mu_y = 6200km \\ \sigma_x^2 = 400^2 & \sigma_y^2 = 300^2 \end{array}$$

- A businessman who purchased tires in bulk selected 100 tires from each type to test whether there is a difference between the life times of two types of tiers.
- It is required to calculate the probability that the difference of the life time of two types of tires is
 - grater than 1250 *km*.
 - 1100 *km*.
- It is also required to decide the size of the sample that express the difference of life time of two types of tires is grater than 1000 *km*. at 95% confidence.
- Give guidance to students to identify the following facts.
- To identify the population characteristics
- To write the distribution of population characteristics
- To write the distribution of difference of two sample means

- Guide the students to solve the problem given using the formula

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

- If any assumptions were made in the solving problems describe them.
- Give an example for a situation where the sum of means of two populations is relevant.
- Guide the group of students for a collective and constructive presentation.

Guidelines to understand the subject matters:

- The difference of the sample means of random samples from two normal populations has a distribution with the following characteristics.

$$\bar{X} - \bar{Y} \sim N\left[(\mu_x - \mu_y), \left(\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}\right)\right]$$

- If the two populations are not normal but two sample sizes are sufficiently large, the distribution of the difference of two sample means has a following distribution according to the Central Limit Theorem.

$$\bar{X} - \bar{Y} \sim N\left[(\mu_x - \mu_y), \left(\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}\right)\right] \text{ approximately}$$

- Sampling distributions can be used to calculate the probability of having difference between sample means and the following statistic is used for this task.

$$Z = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$$

Competency Level 6.6 : Constructs sampling distribution of the sample proportion P

Number of periods : 08

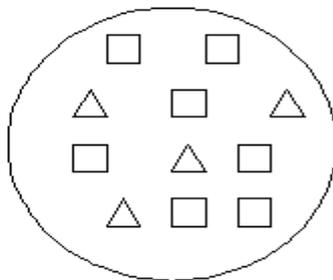
Learning Outcomes :

- Explains the sample proportion and population proportion.
- States the relationship between the ample proportion and population proportion.
- Calculates the variance of the sample proportion.
- Constructs the sampling distribution of the sample proportion.
- Demonstrates the capability of making decisions based on proportion.

Learning - Teaching Process:

Approach

- Draw the attention of students to the questions given with the following diagram.



- What is the proportion of symbols \triangle of the all symbols of the diagram?
- Select any random sample of symbols of size 8 you wish from the above diagram write down the proportion of \triangle symbols.
- Lead a discussion highlighting the following facts.
- The value obtained for the number of units with a certain characteristics of the population divided by the total number of units of the population is called the population proportion.

- The value obtained for the number of units with a certain characteristics in the sample divided by the total number of units in the sample is called sample proportion.
- It is useful to study the sampling distribution of sample proportion of characteristics related to all samples drawn from the population.

Proposed Instructions for learning:

- Divide class in to three groups.
- Allocate the sample sizes that should be taken from finite and infinite population according to the plan given in the following table.

| | First group | Second group | Third group |
|---------------------|-------------|--------------|-------------|
| Finite population | $n = 2$ | $n = 3$ | $n = 4$ |
| Infinite population | $n = 50$ | $n = 100$ | $n = 400$ |

Case 1

- Present a list of business firms to the class.
- Give instructions to student groups to select all possible samples of sample size received by each group considering the list of names as the population.
- Direct the students to express the proportion of boys in each sample.
- Instruct the students to calculate the mean of the sampling distribution of the above sample proportion.
- Direct the students to find the population proportion of boys (π).
- Give an opportunity to identify the relationship between the sample proportion and population proportion.
- Calculate the variance of the sample proportion using the formula $\sigma_p^2 = \frac{\pi(1-\pi)}{n} \left(\frac{N-n}{N-1} \right)$

and calculate the variance and standard deviation of the sampling distribution of P .

Case 2

- Suppose a large number of employees work in that establishment. From all the employees 40% is male.
- Considering each sample size assigned for infinite population direct the students to find the probability that male proportion is
- Less than 0.3
- Between 0.25 and 0.35
- Greater than 0.45
- Prepare for a collective and constructive presentation with your findings before the entire class.

Guidelines to understand the subject matters:

- Since population variable from which the sample is obtained is not normal for a small sample, sampling distribution of the sample proportion is binomial and for a large sample, according to the Central Limit Theorem it is approximately normal.
- Sufficiently large samples should be obtained to consider the sampling distribution of the sample proportion as approximately normal.
- The mean of the sampling distribution of the sample proportion is the same as population proportion ($\mu_p = \pi$).
- When the population from which samples are obtained is infinite the variance of the sampling distribution of the sample proportion can be computed using the formulae
$$\sigma_p^2 = \frac{\pi(1-\pi)}{n}$$
- For a finite population this formulae is
$$\sigma_p^2 = \frac{\pi(1-\pi)}{n} \left[\frac{N-n}{N-1} \right]$$
.
- According to the Central Limit Theorem the sampling distribution of the sample proportion is approximately
$$P \sim N\left(\pi, \frac{\pi(1-\pi)}{n}\right)$$
.
- In solving problems related to sample proportion the normal distribution should be transformed into standard normal distribution.

Competency Level 6.7 : Constructs sampling distribution of difference of two sample proportions for making decisions.

Number of periods : 08

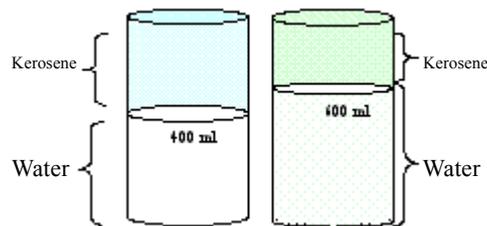
Learning Outcomes :

- Introduces the sampling distribution of difference of two sample proportions
- Calculates the mean and the variance of difference of two sample proportions
- Constructs the sampling distribution of difference of two sample proportions
- Solve problems related to the difference of proportions of random samples taken from two populations.
- Demonstrates the preparedness to understand the differences of various proportions come across in the life.

Learning - Teaching Process:

Approach

- Demonstrate to the students two beakers sized 800 ml filled with different volume of water and kerosene as shown in the following diagram.



(For this task, assistance can be taken from the teacher in charge of the laboratory or any type of two liquids may be used.)

- Lead the discussion highlighting the following facts.
 - The proportion of the kerosene of the first beaker is 0.5
 - The proportion of the kerosene of the second beaker is 0.25
 - The proportion of the kerosene of the first beaker is higher than 0.25 that of second beaker.
 - This can be identified as the difference of two proportions.

- If the difference of two proportions of a characteristic of all samples sized n_1 and n_2 taken from different population is $P_1 - P_2$, the probability distribution of $P_1 - P_2$ is named as the sampling distribution of the difference of two sample proportions.
- Exploration of special features of this sampling distribution is useful

Proposed Instructions for learning:

- Divide the class in to three small groups.
- Assign following explorations and make them engage activities.

| First population | Second population |
|--------------------------------------|--------------------------------------|
| Geetha, Mala, Leela Sarath, Nihal | Neela, Manel, Ruwan Jagath, Amara |

- Make students engage in activities with the sample size given bellow
- Sample size 2
- Sample size 3
- Sample size 4
- Denote the all sample from first population as S_1 and all samples from second population as S_2 .
- What is the value of proportion of girls, π_1 in the first population?
- What is the value of proportion of girls π_2 in the second population?
- What is the value of difference of proportion $\pi_1 - \pi_2$?
- Denote the proportion of girls in the samples taken from first population as P_1 and the proportion of girls in the samples taken from second population as P_2 .
- Fill the following table.

| Sample Number | Sample units of the first population | P_1 | Sample units of the second population | P_2 | $P_1 - P_2$ |
|---------------|--------------------------------------|-------|---------------------------------------|-------|-------------|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |
| 7 | | | | | |
| 8 | | | | | |
| 9 | | | | | |
| 10 | | | | | |

- Complete the distribution of $P_1 - P_2$ summarizing the above table.

| | |
|-------------|--|
| $P_1 - P_2$ | |
| Probability | |

- Obtain the mean, $\mu_{P_1 - P_2}$ of the distribution of $P_1 - P_2$.
- What you can say about the values of $\pi_1 - \pi_2$ and $\mu_{P_1 - P_2}$.
- Obtain the mean of the distribution of $P_1 - P_2$.
- Substituting the values that you have obtained for π_1 and π_2 to the following formulae, obtain the variance of the sampling distribution of difference of two sample proportions.
- $$\sigma_{P_1 - P_2}^2 = \frac{\pi_1(1 - \pi_1)}{n_1} + \frac{\pi_2(1 - \pi_2)}{n_2}$$
- Verify that the variance of distribution of $P_1 - P_2$ is equal to the value obtained from above formulae.
- Suppose that 10% of the employees of institute A are trainees and the 7% of the employees of institute B are trainees. When a sample of 100 employees from the institute A and a sample of 120 employees from the institute B are obtained, find the probability that the difference of proportions of trainees of two institutes is less than 2%.

- Prepare for a collective and constructive presentation with your findings before the entire class.

Guidelines to understand the subject matters:

- The mean of the distribution of difference of two sample proportions is equal to the difference of respective populations proportions. ($\mu_{P_1-P_2} = \pi_1 - \pi_2$).

- The variance of the sampling distribution of the difference of two sample proportions is given by following formulae.

- $$\sigma_{P_1-P_2}^2 = \frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2}$$

- What ever the nature of the two proportions if the sample size is sufficiently large, the distribution of the difference of two sample proportions has an approximately normal distribution according to the Central Limit Theorem.

- In solving of problems related to the difference of two sample proportions, the following formula is used to transform the sampling distribution of $P_1 - P_2$ to the standard normal distribution.

- $$Z = \frac{(P_1 - P_2) - (\pi_1 - \pi_2)}{\sqrt{\frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2}}}$$

Competency Level 6.8 : Uses point estimation for making business decisions.

Number of periods : 13

Learning Outcomes :

- Explains the statistical Estimation.
- State the properties of a good Estimator.
- Computes the standard error of selected estimators.
- Behave logically in making optimum decisions.
- Makes more realistic conclusions considering variations.

Learning - Teaching Process:

Approach

- Presents the following flow chart to the class which shows the process of making conclusions about the population by selecting a part of the population.



- Lead a discussion highlighting the following facts.
- The arrows indicate the process and rectangles indicate the resulting outcomes.
- The functions of samples use to make conclusions about population parameters are known as statistics.
- The process of making conclusions using statistics is the estimation.
- The statistics used to estimate a parameter is an estimator.
- The following are some examples for estimator.
 - Sample Mean
 - sample variance
 - sample proportion
- The value obtained for the estimator using sample data is called an estimate.
- Estimation of the parameter by an estimate is point estimation.
- The estimator used in point estimation should be better than any other possible estimators.

Proposed Instructions for learning:

- Allocate the following two cases to groups of students
 - The head of an institution with 50 workers intends to make an assessment about number of late attendance using time of arrivals recorded in a newly fixed machine to record attendance.
- Last week 5 workers came late and the times in minutes for their late arrivals are given below. 2, 1, 4, 3, 5
- From the above values all possible samples of size 3 were taken. The human resource manager wants to estimate the mean time of late arrivals and its spread and prepare a report about the behavior of late arrivals of workers.

- If the sample observations are y_1, y_2, y_3

$$\bar{y}_1 = \frac{y_1 + y_2 + y_3}{3}, \quad \bar{y}_2 = \frac{y_1 + y_2 + y_3}{3}, \quad \bar{y}_3 = \frac{y_1 + y_2 + y_3}{3}$$

- An outlet of “Pizza” distribution intends to guarantee “Pizza” delivery to home within 10 minutes. The followings are the delivery times in minutes according to the demand 2, 5, 25, 8
 - The all possible samples of size 3 are taken from these values. It is intended to estimate the mean and its spread using the best estimator among point estimators and use it in advertising
- If x_1, x_2, x_3 are sample observations

$$T_1 = \frac{x_1 + x_2 + x_3}{3}, \quad T_2 = \frac{x_1 + 2x_2 + x_3}{4}, \quad T_3 = \frac{x_1 + x_2 + 2x_3}{4}$$

- Give an opportunity to find out the characteristics of estimators following the instructions given below.
- Name the estimators for population mean, variance and proportion relevant to the given case.
- What are characteristics that should be considered to assess an estimator for population parameter?
- Verify whether the estimators you have selected have the characteristics of a good estimator using the given data.
- Examine whether the estimators suggested by the institution have the characteristics of a good estimator you stated and whether these estimators can or cannot be accepted.
- Calculate the standard error of the best estimator you have selected for estimation of the population parameter in the case assigned to you.

- Explain whether the estimation of a population parameter by a single value is successful.
- Prepare for a collective and constructive presentation with your findings before the entire class.

Guidelines to understand the subject matters:

- The following summary measures which are calculated using sample data are point estimators.

- Sample Mean \bar{X}
- Sample Median M_d
- Sample Mode M_o
- Sample Variance S^2
- Sample Standard Deviation S
- Sample Proportion P

- A good estimator for a population parameter should possess the following characteristics.

- Unbiasedness
- Efficiency
- Consistency
- Sufficiency

- When the expected value of estimator is equal to the population parameter, that estimator is an unbiased estimator.

- The unbiasedness of a parameter can be defined as follows.

- If $\hat{\theta}$ is an unbiased estimator of θ and if $E(\hat{\theta}) = \theta$, then $\hat{\theta}$ is an unbiased estimator for θ .
- For an unbiased estimator $E(\hat{\theta}) - \theta = 0$
- $E(\hat{\theta}) - \theta$ is the bias of estimator $\hat{\theta}$.
- Since $E(\bar{X}) = \mu$ sample Mean \bar{X} is an unbiased estimator for μ

- The following estimator is an unbiased estimator for population variance and it is known as sample variance.

- $$S^2 = \frac{\sum (X - \bar{X})^2}{n - 1} = \frac{1}{n - 1} (\sum X^2 - n\bar{X}^2)$$

- Sample median is an unbiased estimator for population median and sample proportion is an unbiased estimator for population proportion.
- The estimator with the minimum variance is the most efficient estimator among two or several estimators for a parameter.

- The reciprocal of the variance of an estimator is a measure of efficiency of an estimator.
- When the population is normal the variance of sample mean and variance of sample median are given below.

- $Var(\bar{X}) = \frac{\sigma^2}{n}$

- $Var(M_n) = \frac{\pi\sigma^2}{2n}$

- $\frac{\sigma^2}{n} < \frac{\pi\sigma^2}{2n}$

- Thus the variance of the unbiased estimator sample mean is less than the variance of the sample median.
- A consistent point estimator should satisfy the following conditions
 - As the sample size increases the expected value of the estimator approaches the parameter

$$\lim_{n \rightarrow \infty} E(\hat{\theta}) = \theta$$

- As the sample size increases the variance of the estimator approaches zero.

- $\lim_{n \rightarrow \infty} Var(\hat{\theta}) = 0$

- When all information about the parameter in the sample is used in calculating an estimator such an estimator is a sufficient estimator.
- Since the sample mean contains all information in the sample about the population parameter sample mean is a sufficient estimator.

Competency Level 6.9 : Uses interval estimation for making business decisions related to population mean.

Number of periods : 15

Learning Outcomes :

- Defines Confidence Intervals.
- Calculates Confidence Intervals for population mean related to a given confidence level.
- Explains success and problems of interval estimation.
- Makes optimal decisions using appropriate principles..
- Uses simple models for calculations.

Learning - Teaching Process:

Approach

- Presents following statements to the class.
 - If 100 boxes are inspected from a lot of boxes of matches, 95 boxes contain sticks 48 to 53 in a box.
 - There is a 95 % confidence that average life time of light bulbs produced by this institute is between 900 hrs. and 1100 hrs.
- Lead the discussion highlighting the following facts.
 - An interval of values which can contain the relevant parameter is given by both statements above.
 - The interval of values is expressed with a certain level of probability.
 - This probability level is identified as the confidence level.
 - The confidence interval can be constructed for the parameter using sampling distribution of the statistics and its' standard error.

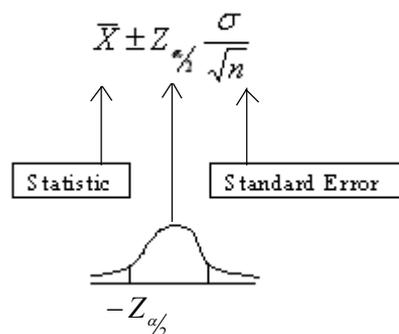
Proposed Instructions for learning:

- Describe the following cases among the group of students.
 - A credit card company expects to decide about the average monthly balance of credit card holders. It was revealed that the mean and the standard deviation of the monthly balance of the card holder are 1100 rupees and 156 rupees respectively in the inspection of random sample with 50 accounts.

- It is known that weight of marbles produced by a company distributes normally with standard deviation 25 g. The statistics calculate from a random sample of 36 marbles obtained for the purpose of making a decision about the mean weight, are as follows.

$$\bar{X} = 740g \quad SD = 30g$$

- Owner of a filling station wants to make a decision about the number of consumers come to the station per day. It is found that thirty vehicles have come to the station per day within a period of 60 days.
- Guide the students to construct interval estimates, giving following instructions.
 - Express the distribution of population and sampling distribution of the sample mean for the case that you have obtained.(State the assumptions if necessary.)
 - Calculate the standard error of the sample mean.
 - Construct the 95 % confidence interval for the population mean using the given formula or another method.
 - Construct the 99 % confidence interval for the population mean.
 - Express the decision about the population mean from each of the confidence interval that you have calculated.
 - Give your ideas about the width of the confidence level of the two confidence intervals.
 - What extent the decision about the parameter μ is precise when the width of the confidence interval increases?
 - Explain the relation between the precision and the confidence of the confidence intervals constructed for the estimation of parameters.
 - Further confirm your decision calculating confidence intervals with a least level such as 50% and a higher level such as 98% relevant to the case that you have received.
- Prepare for a collective and constructive presentation with your findings before the entire class.
 - Formula for the $(1 - \alpha)\%$ Confidence interval for mean μ of a normal population with known variance.



Guidelines to understand the subject matters:

- The interval estimation is making an interval for the true value of the parameter.
- The interval that contains population mean can be obtained relevant to a certain confidence level.
- The confidence level is denoted by $(1 - \alpha)\%$ and generally 95% and 99% are used.
- Deciding of the sampling distribution of the statistics which is used to estimate the parameter is necessary.

- The $(1 - \alpha)\%$ confidence interval for the mean μ of a normal population with known variance is expressed as follows.

$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

- At the higher confidence levels, width of the confidence interval increases.
- By increasing the size of the sample, the precision of the confidence interval can be increased up to a certain level under the given confidence level.

Competency Level 6.10 : Uses the interval estimation for the difference between two population means for making business decisions.

Number of periods : 15

Learning Outcomes :

- Explains the importance of estimation of the difference between two population means.
- Computes the standard error for the difference between two population means.
- Computes the confidence intervals for the difference between two population means.
- Computes common variance for the difference between means using separate variances.
- Demonstrates the skills for making decisions comparing cases.

Learning - Teaching Process:

Approach

- Present the following ideas to class.
- The section head teacher wants to decide which class is better according to the marks of students of classes A and B of grade 12.
- Draw the attention of students to the following question.
- If you have opportunity to help the teacher to make this decision what is the statistical techniques you suggest to her? Explain the reason.
- **Lead the discussion highlighting the following facts.**
- It is required to consider the difference between random variables in making decisions related to business fields.
- Confidence intervals can be calculated to estimate the difference between population means.
- In calculating this confidence intervals the relevant sampling distribution should be based.

Proposed Instructions for learning:

- Present the following cases and two confidence levels to group of students.
- 95% confidence level
- 99 % confidence level
- The manager of a supermarkets believes that the sales of goods arranged in racks below the eye level is high and the sales of goods in racks above eye level is low. It is known that the number of goods selected from each rack has normal distribution. Two samples each of size 50 customers were examined. From these samples the following statistics were observed.

Sales from racks below eye level

$$\bar{x}_1 = 3.05$$

$$n_1 = 50$$

$$\sigma_1^2 = 0.5$$

Sales from racks above eye level

$$\bar{x}_2 = 3.5$$

$$n_2 = 50$$

$$\sigma_2^2 = 0.5$$

- A certain metal is produced by a standard method. A new production method has been developed by adding a certain mixture to metal. The producers want to examine whether there is true difference between mean weights bearable by the metal produced by these two methods.

| | | | | | | | | | | | | |
|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Standard Method | 428 | 419 | 458 | 439 | 441 | 456 | 463 | 429 | 438 | 445 | 441 | 463 |
| New method | 462 | 448 | 435 | 465 | 429 | 472 | 453 | 459 | 427 | 468 | 452 | 447 |

- Copper pieces are produced using copper dust in a production factory without evaporation under a controlled situation. The strength of such copper bars were measured. For a sample size $n_1 = 4$, the mean strength was $\bar{x}_1 = 0.22$ and the variance was $s_1^2 = 0.001$. The mean strength of copper pieces produced in another factory using the same method for a sample of size 5 was $\bar{x}_2 = 0.17$ and variance was $s_2^2 = 0.002$.
- Give the following instruction to group of students.
 - State any assumptions required to estimate the mean difference relevant to each problem.
 - Compute the standard error of the mean difference relevant to each problem.
 - Estimate the confidence interval for the mean difference with the confidence level received by your group for each problem.
 - Comment on the difference between two sample means.
- Prepare for a collective and constructive presentation with your findings before the entire class.

The formulas for solving problems

- The $(1 - \alpha)\%$ confidence interval for the difference between population means $\mu_1 - \mu_2$ when the variances of two normal population are known is

$$\bar{x}_1 - \bar{x}_2 \pm Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

- The $(1 - \alpha)\%$ confidence interval for the difference between population means $\mu_1 - \mu_2$ when the small samples are taken from independent normal populations with unknown but equal variance:

$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2, (n_1+n_2-2)} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

- Since the common variance S_p^2 is unknown it should be calculated with the sample variances using the formula.

$$S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Competency Level 6.11 : Uses the interval estimation for making business decisions related to population proportion.

Number of periods : 4

Learning Outcomes :

- Express condition for the construction of confidence intervals for the population proportion.
- Writes the formula used in constructing the confidence intervals for the population proportion.
- Calculates the confidence intervals for the population proportion based on given data.
- Makes future plans using estimated population proportion.
- Demonstrates the preparedness to behave minimizing variations of estimates that are based on sample data.

Learning - Teaching Process:

Approach

- Present the following statements to the class and lead the discussion highlighting the following facts.
 - A sample survey has confirmed that the party A will win the next general election.
 - A survey consists of sample of cricket fans predicts with 95% confidence that Sri Lanka will win the next world cricket Cup.
- Such prediction can be made about the future events related with proportions.
- The sample proportions can be used to express the predictions.
- The following formula is used to construct the confidence intervals for population proportion π using sample proportion 'P' approximately.

$$P \pm z_{\alpha/2} \sqrt{\frac{P(1-P)}{n}}$$

Proposed Instructions for learning:

- Divide the class in to three small groups.
- Guide each group to solve problems with given confidence levels.
 - 95% confidence level
 - 98% confidence level
 - 99% confidence level

- A pre election survey conducted using a random sample of 100 registered voters revealed that 55 voters were in favor of the candidate A. Construct a confidence interval for the percentage of voters favored to the candidate A in the election.
- Relevant to the above problem find the size of the sample to state with the confidence level given to you that this candidate will win the election.
- When the coin is tossed 400 times, the head is obtained 260 times. Construct a confidence interval for the proportion of obtaining the head when ever this coin is tossed unlimited number of times.
- The Confidence interval constructed for the proportion of defects produced by a certain factory is 0.313 – 0.407. The proportion of defective items in the sample is 0.36. Find the size of the sample under the confidence level given to you.
- Prepare for a collective and constructive presentation with your findings before the entire class.

Guidelines to understand the subject matters:

- When the sample size n taken from a population with population proportion π , is large, the distribution of sample proportion P is approximately as follows ,

$$P \sim N \left[\pi, \frac{\pi(1-\pi)}{n} \right]$$

- Then
$$Z = \frac{P - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} \sim N(0,1)$$

- Central Limit theorem is the bases for above transformation.
- It is unable to assume that the sampling distribution of P is a normal distribution when the sample size is small.
- The following formula is used to construct the $(1 - \alpha)\%$ confidence interval for the population proportion π

$$P \pm z_{\alpha/2} \sqrt{\frac{\pi(1-\pi)}{n}}$$

- The sample proportion P can be substituted for π on a good estimator because π is generally unknown and sample size is large.
- Then the following formula is used to construct the confidence interval for π

$$P \pm z_{\alpha/2} \sqrt{\frac{P(1-P)}{n}}$$

- For taking optimal decisions in the business and social fields, this method is most suitable to estimate the population proportion.

Competency Level 6.12 : Estimates intervals for making business decisions related to difference of two population proportions.

Number of periods : 4

Learning Outcomes :

- Explains the importance of estimation of difference of two population proportion.
- Calculates the standard error of the difference of two population proportions relevant to the situation.
- Calculates the confidence intervals for the difference of two population proportions.
- Makes future plans using the difference of two population proportions.
- Takes decisions comparing two characteristics.

Learning - Teaching Process:

Approach

- Present the following to the class.
“80% of the housewives of district A and 98% of the housewives of district B have expressed the desire for washing cloths using detergent powder”
- Lead the discussion highlighting the following facts.
 - The difference of proportion of random variables should be considered for making decisions.
 - The confidence intervals can be calculated for estimation of the difference of population proportions.
 - The relevant sampling distribution should be used in the construction of confidence intervals.

Proposed Instructions for learning:

- Provide the following levels of confidence to group of students.
 - 95% confidence level
 - 98% confidence level
 - 99% confidence level
- Describe the following learning opportunities among the groups of students.
- For the purpose of comparing newly found two types of insecticides 1000 of insects were enclosed in to each two equal size rooms and sprayed type A to one room and type B to the other room. It was found that 900 insects were dead in the room where A type was sprayed and 820 insects were dead in the room where B type was sprayed. Estimate the confidence interval for the difference in efficiency of two types of insecticides to destroy the insects.

- Two types of bolts are produced by two machines P and Q. 200 bolts selected by each machines were tested. 15 bolts produced by machine P and 5 bolts produced by machine Q were defectives. Estimate the confidence intervals for the difference of proportions of defective bolts produced by both machines.
- Provide following instructions to the group of students.
- If there are assumptions for each case, state them.
- Calculate the standard error of the difference of proportions
- Construct the confidence intervals relevant to the confidence level that you are given.
- Prepare for a collective and constructive presentation with your findings before the entire class.

The $(1 - \alpha)\%$ confidence interval used for solving problems

$$(P_x - P_y) \pm Z_{\alpha/2} \sqrt{\frac{P_x(1-P_x)}{n_x} + \frac{P_y(1-P_y)}{n_y}}$$

Guidelines to understand the subject matters:

- When X and Y are two random variables distributed as binomial distribution with parameters π_1 and π_2 according to the central limit theorem the difference of two proportions distributes approximately normally as follows.

$$P_x - P_y \sim N\left[\left(\pi_x - \pi_y\right), \frac{\pi_x(1-\pi_x)}{n_x} + \frac{\pi_y(1-\pi_y)}{n_y}\right]$$

- The standard normal distribution can be obtained as follows

$$Z = \frac{(P_x - P_y) - (\pi_x - \pi_y)}{\sqrt{\frac{\pi_x(1-\pi_x)}{n_x} + \frac{\pi_y(1-\pi_y)}{n_y}}} \sim N(0,1)$$

- Since the population proportions are unknown, sample proportions are used instead.
- The $(1 - \alpha)\%$ confidence interval for the difference of two population proportions is

$$(P_x - P_y) \pm Z_{\alpha/2} \sqrt{\frac{P_x(1-P_x)}{n_x} + \frac{P_y(1-P_y)}{n_y}}$$

Competency Level 6.13 : Explain the process of testing of statistical hypothesis.

Number of periods : 10

Learning Outcomes :

- Defines the null hypothesis and alternative hypothesis.
- Explains the terms used in the process of testing hypothesis.
- Constructs hypothesis relevant to a given problem.
- Demonstrates the capability of solving problems minimizing the errors.

Learning - Teaching Process:

Approach

- Call two students before the class and ask them to present the following dialog.
- (A food inspector came to a bakery and had the following dialog with the baker after establishing the identity.)

Food Control Inspector : Hello, Let's have a loaf of bread weighted.

Baker : Ok sir, all these are 450 grams.

Food Control Inspector : Look at this, this is only 440 grams.

Baker : One or two loaf may change. You can't have a perfect scale to weigh to nearest decimal.

Food control Inspector : See this one. It is only 435 grams. For fareness I suggest we should have a test to see whether we have evidence to accept your claim.

(Food control Inspector selects a sample from the lot and note down the weight of each bread in the sample)

Food Control Inspector : From these observations I can tell you whether we can accept or reject your claim. I will tell you next time.

Baker : Ok Sir, I do not make bread less than 450 grams saying that my breads weigh 450 grams.

- Lead the discussion highlighting the following facts.
 - The baker is adamant in claiming that the bread he produced weight 450 grams.
 - An opinion, a claim, a belief of a person about something can be called a hypothesis.
 - Baker always expressed ideas about the entire production of breads.
 - The initial hypothesis constructed about the population is null hypothesis.

- It is denoted by H_0 :
- The hypothesis constructed against the null hypothesis is called alternative hypothesis.
- It is denoted by H_1 :
- It is possible to use evidence obtained from the sample of food inspector to decide whether to accept or reject the claim of the baker about the weights of the breads.

Guidelines to understand the subject matters:

- Lead the discussion highlighting the following facts.
 - There are four possible decisions in hypothesis testing
 - Not rejecting the null hypothesis (H_0) when it is true is a correct decision.
 - Rejection of the null hypothesis (H_0) when it is true is type one error.
 - Not rejecting the alternative hypothesis (H_1) when it is true is a correct decision.
 - Rejecting the alternative hypothesis (H_1) when it is true is type two error.
 - These four decisions can be presented as in the following table

| | H_0 : true | H_0 : false |
|-----------------------|-----------------------------------|----------------------------------|
| Reject H_0 : | Type one error (α) | Correct decision ($1 - \beta$) |
| Do not Reject H_0 : | Correct decision ($1 - \alpha$) | Type two error (β) |

- In testing hypothesis the probability of making type two error (β) is minimized keeping the probability of making type one error at the constant level α and α is called the significance level of the test of hypothesis.
- The significance level is known as the size of the test.
- $(1 - \beta)$ or accept H_1 when it is true is the power of the test.

Competency Level 6.14(a) : Tests hypothesis related to population mean and population proportion to make business decisions.

Number of periods : 4

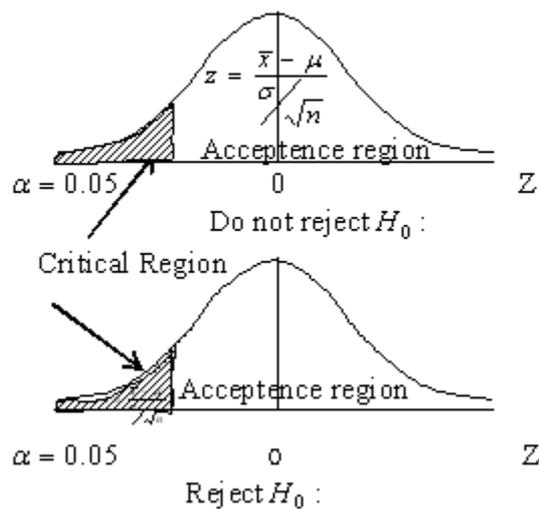
Learning Outcomes :

- States the testing of a hypothesis.
- Tests the hypothesis related to a given problem using appropriate techniques.
- Makes business decisions based on the technique of tests of hypothesis.
- Solves problems come a cross logically using hypothesis.
- Gathers evidence to establish the hypothesis.

Learning - Teaching Process:

Approach

- Demonstrate the following diagrams to the class.



- Lead a discussion highlighting the following facts
 - The normal distributions are shown by above diagrams.
 - The shaded area in the diagrams is named as the critical region and the other area is the acceptance region.
 - To distinguish these two regions, the z value relevant to the significance level is used.
 - To make decisions based on hypothesis, the technique of statistical test of hypothesis is to be followed.

Proposed Instructions for learning:

- Divide the students into two groups and assign them each of the following problems to find solutions
- Suppose that the daily production of number of biscuit packets produced by a company is normally distributed with mean 20 000 and standard deviation 1800. The organizational structure of company was changed in order to raise the production level. Under this new organizational structure the mean of the daily production over a period of 81 days was 20300. Test whether the production level of the company has increased due to organizational change at $\alpha = 0.05$ level.
- Suppose that the time required for a bus to travel between Maharagama and Pettah on 138, route has a normal distribution. Before constructing the flyover at Nugegoda the mean time required for this bus to reach the destination was 48 minutes. After constructing the flyover in a normal working day the mean time for 64 busses for this journey was 40 minutes with the standard deviation 4.4 minutes. Test whether the mean time to reach the destination on this route has changed due to the construction of flyover at 0.05 level.

Competency 7.0 : Explores and forecasts the time dependent variables
Competency Level 7.1 : Explains the variations of the time dependent variables
No of Periods : 04

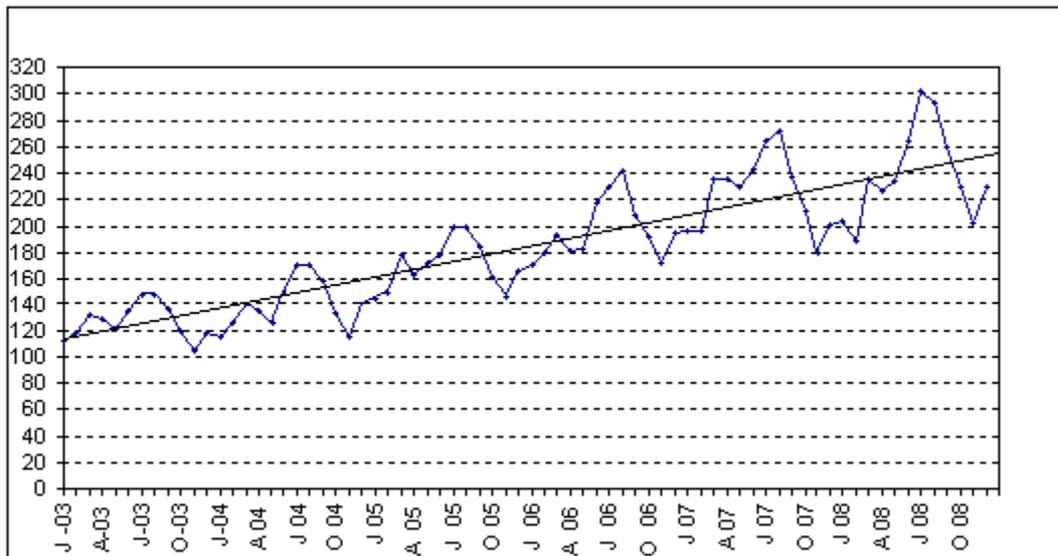
Learning Outcomes :

- Names and Explains the fluctuations of the time dependent variables.
- Identifies the fluctuations of data that vary with the time by observing the graph
- Presents the provisions to get the future decisions by finding the reasons of those fluctuations.
- Makes decisions regarding the future by studying the past and present experiences.
- Uses the graphs to simplify the complexity.

Learning -Teaching Process :

Approach :

- Show the graph given below before the class which gives the number of tourists arrival at the Colombo international air port, Sri Lanka from 2003 to 2008



- Discuss and highlight the following points.
 - No of arrivals are represented according to the time by the Graph.
 - Data that had been collected in successive periods were used to construct the graph.
 - The graph which shows the variations of variables with the time is called a time series graph.

Proposed Instructions for learning

- Distribute the following tables among three groups.

Table - 1
No of infant's food packets sold in the supper Market
(In Thousands from 1997 to 2008)

| Years | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 |
|----------------|------|------|------|------|------|------|------|------|------|------|------|------|
| No. of Packets | 15 | 16 | 17 | 21 | 22 | 21 | 19 | 23 | 28 | 25 | 26 | 30 |

Table - 2
No. of Foreigners stayed in a Tourists Hotel (In Thousands)

| Quarters Years | I | | | |
|-------------------|----|----|-----|----|
| | I | II | III | IV |
| 2004 | 19 | 20 | 23 | 25 |
| 2005 | 18 | 17 | 21 | 26 |
| 2006 | 18 | 16 | 22 | 28 |
| 2007 | 21 | 20 | 24 | 31 |
| 2008 | 23 | 21 | 23 | 34 |

Table - 3
Mean annual rainfall in Sri Lanka (in mm s from 1993 to 2008)

| Year | Mean annual rainfall mm | Year | Mean annual rainfall mm |
|------|-------------------------|------|-------------------------|
| 1993 | 30 | 2001 | 28 |
| 1994 | 28.5 | 2002 | 29 |
| 1995 | 28 | 2003 | 30 |
| 1996 | 29 | 2004 | 31 |
| 1997 | 30 | 2005 | 30.3 |
| 1998 | 31 | 2006 | 29 |
| 1999 | 30.5 | 2007 | 28 |
| 2000 | 29 | 2008 | 28.6 |

- Lead the students to draw a graph using the given data in the table.
 - Guide the groups to identify the fluctuations of the variables by observing the graph.
 - Prepare the students to explain the components that are mostly represented by time series.
 - Get the students to find out as many as possible variations of time series and the causes for it.
 - Get them to point out facts when observing a time series along with the following topics.
 - Collecting data for long periods.
 - Collecting data for successive periods.
- Prepare for a collective and constructive presentation with your findings before the entire class.

Components of Time Series :

Secular trend

Generally the long – term movements and the direction of the time series is explained by the secular trend. It may be an increasing, decreasing or constant situation. As examples,

- (i) Increasing demand of the computer devices over past years.
- (ii) Decreasing of the birth rate of Sri Lanka over several past decades.

Whether the long-term trend is a straight line or a curve, it can be identified by the graph. The following techniques can be used to estimate the trend

- i. Freehand Method
- ii. Semi Averages Method
- iii. Least Square Method
- iv. Moving Averages Method

Seasonal Variation

Short-term variations that happen in regular intervals repeatedly with same amount are called seasonal variations **Eg:**

- (i) Increasing sales of cloths in every month of April
- (ii) Decrease of the price of rice after harvesting in ‘Maha’ Season.

Following techniques can be used for estimating the Seasonal variation in the time series

- i. Average percentage Method
- ii. Ratio to trend Method
- iii. Ratio to moving average Method

Cyclical Variation

The cyclical variations are generally long term fluctuations that occur repeatedly about the trend line. It has been 10, 15, 20 years of periods. Booms and recessions are some examples for business cycles in the business fields

Irregular Variations

Unpredictable random variations that occur in short term is called irregular variations. Irregular variations can be recognized by removing other three variations in the time series. Those types of variation can happen due to the events such as following.

- i. Strikes
- ii. Abrupt shortage of raw materials

Guidelines to understand the subject matters:

- The data that varies as a function of the time is known as time series.
- Time series data can be represented by a graph.

- There are some fluctuations in time series graph and the several factors relevant to the field affect those fluctuations.

- Strikes, Public Policies, Technological inventions, Droughts etc. could be identified as the causes that affect the variations of the time series.

- Variations of a time series can be divided in to four components as follows.
 - Secular trend
 - Seasonal Variations
 - Cyclical Variations
 - Irregular Variations

Competency Level 7.2 : Uses the most relevant model to separate the components of Time Series

No of Periods : 02

Learning Outcomes :

- Identifies the components of Time Series.
- States the Models that should be used to analyze the Time Series.
- Decides the relevant Model of Time series for the given situation.
- Shows the preparedness to analyze a Time Series.
- Applies Models to simplify the Complexity.

Learning -Teaching Process:

Approach:

- Present the given models to the class and discuss and highlight the following points.
 - i. $Q_d = a - bp$
 - ii. $Q_s = a + bp$
 - iii. $Y = C + I + G + (X - M)$
 - iv. $Y = TCSI$
 - v. $Y = T + C + S + I$
 - vi. $Y = TC + SI$
- The relationships between the two variables given below is represented by the models which are shown above (i), (ii) and (iii)
 - Price and demand of a good
 - Price and supply of a good
 - Aggregate income and revenue

Y - National Income
 C - Consumer Expenditure
 I - Investment
 G - Public Expenditure
 X - Export income
 M - Import cost

- The relationships among the components of time series is represented by the last three equations above.

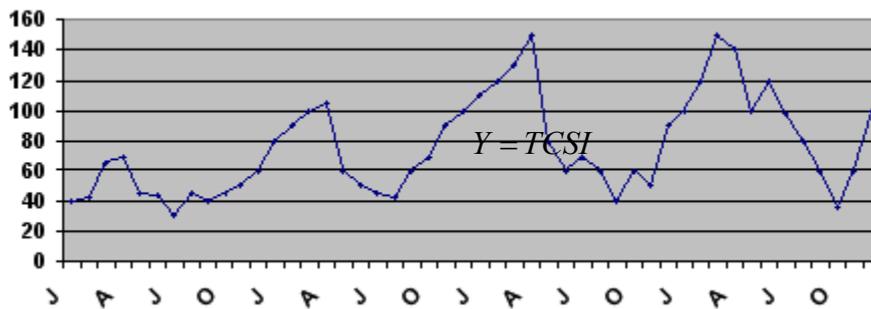
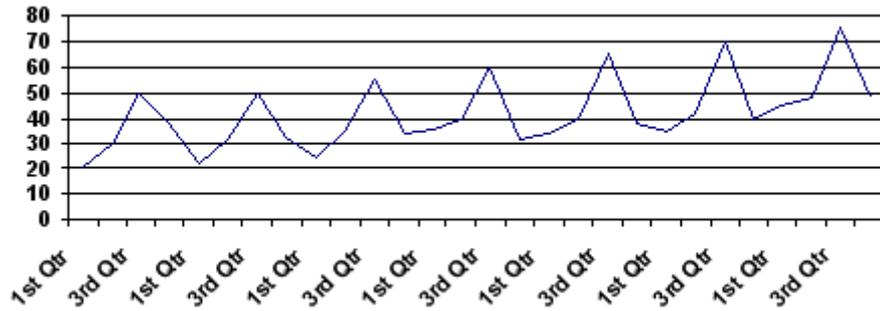
Y - Time Series variable

T - Long term Trend

- S* - Seasonal Variations
- C* - Cyclical Variations
- I* - Irregular Variations

Proposed Instructions for learning:

- Distribute the following graphs between two groups.



- Name a variable, which is suitable for the vertical axis of the graph.
- Direct the students to name the components of time series that are exposed from the graph.
- Inquire the students regarding the interdependence of those components.
- Direct the students to select the most suitable model that is given below for analyzing the graph
 - (i) $Y = T + C + S + I$
 - (ii) $Y = TCSI$
- State the reasons for their answers.
- Direct the students to collect more facts regarding the following themes that should be considered before analyzing the time series.
 - Editing time series data
 - Selecting the model for analyzing

- Direct the students to illustrate the suitable situations where
 - Additive Model and
 - Multiplicative Model can be applied in the field of Business.
- Prepare for a collective and constructive presentation with your findings before the entire class.

Guidelines to understand the subject matters:

- Time series data should be edited before analyzing.
- Adjustment may be needed for calendar variations, price and population changes and other changes, which affect the comparability of the data.
- Next, suitable model should be identified.
- The model can be decided easily, by observing the time series graph.
- When the components are independent, the seasonal pattern become parallel with the Long-term trend.
- When the components are dependent the Multiplicative Model can be applied to analyze the components.
- The multiplicative Model is applied to analyze, because of the components mainly dependent in the field of business,
- When the components are independent, Additive Model is suitable to analyze the time series

Competency Level 7.3 : Uses Free Hand method to calculate the Trend

No of Periods : 02

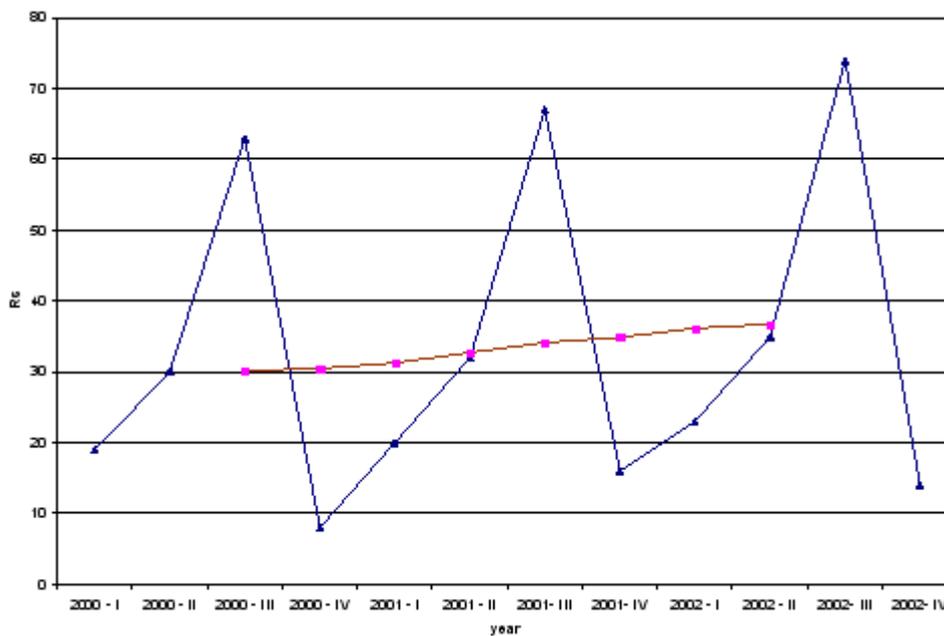
Learning Outcomes :

- Draws the trend line by using the Free Hand method.
- Explains advantages/disadvantages of that method.
- Formulates the equation for the trend line obtained by the Free Hand Method
- Identifies the direction of the change of variable.
- Uses easy and rapid method for making decisions.

Learning -Teaching Process :

Approach:

- Present the following graph to the class.



- Lead a discussion highlighting the following facts
 - The trend can be separated from the time series.
 - The Trend can be obtained as a straight line or a curve on the time series graph.

Proposed Instructions for learning:

- Divide the students into several groups and provide the following table.

Sales Income (in Rupees) of a business

| Years \ Quarters | I | II | III | IV |
|-------------------------|----------|-----------|------------|-----------|
| 2004 | 16 | 21 | 09 | 18 |
| 2005 | 15 | 20 | 10 | 18 |
| 2006 | 17 | 24 | 13 | 22 |
| 2007 | 17 | 25 | 11 | 21 |
| 2008 | 18 | 26 | 14 | 25 |

- Direct the students to draw a graph for the given data.
- Lead them to obtain most suitable trend line drawing a straight line on the graph arbitrarily.
- Let them calculate the equation of the line drawn on the graph using the following steps.
 - Obtain the intercept as the point where the trend line intersects with the vertical axis. (β_0)
 - Calculate the gradient (β_1) by $\frac{\nabla y}{\nabla x}$ considering two points on the trend line.
 - Obtain the estimated equation $\hat{y} = \beta_0 + \beta_1 x_i$ of the trend line by substituting values of β_0 and β_1 .
- Let them to highlight the advantages and disadvantages of fitting the trend line by free hand method.
- Prepare for a collective and constructive presentation with your findings before the entire class.

Guidelines to understand the subject matters:

- Drawing the trend line arbitrarily representing all points on the time series graph is called the freehand method.

- Advantages of obtaining the trend line by using the free hand method are as follows:
 - Easy to draw
 - Flexible (can obtain straight line or curve)
 - Time saving

- Disadvantages of obtaining trend line by using free hand method are as follows
 - Highlight affects by subjective elements.
 - Estimate different trend lines from the same data.

Competency Level 7.4 : Uses the method of Semi-Averages to quantify the trend.

No of Periods : 02

Learning Outcomes :

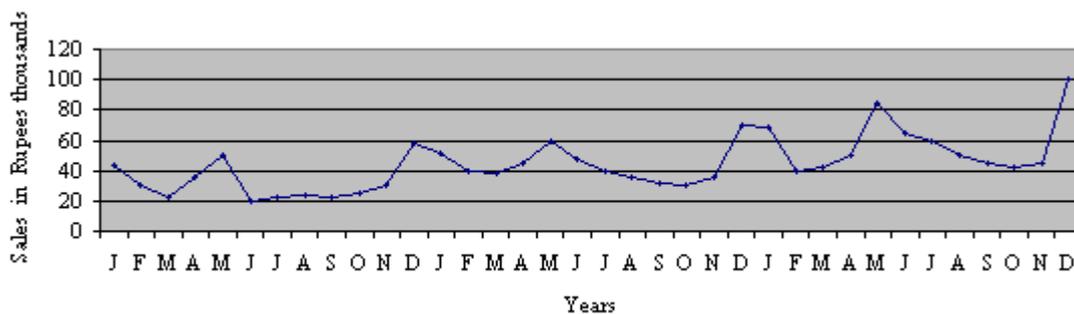
- States the steps for estimating linear trend by using the Semi-Averages method.
- Constructs the linear trend by using the Semi-Averages method.
- States advantages and disadvantages of the above method.
- Identifies the direction of variation in variables easily using simple mathematical technique.
- Separates and identifies the trend using mathematical techniques.

Learning -Teaching Process:

Approach:

- Display the graph given bellow to the class.

**Income in thousands rupees received by a shop
of selling greeting cards.**



- Lead a discussion using the graph highlighting the following points.
- Several trend lines can be drawn using the same set of data.
- One trend line should be found for a particular set of data, because it is difficult to make decisions with several trend lines.

Proposed Instructions for learning:

- Provide the following tables to the two groups

**Table – 1
Sales of the institute A**

| | | | | | | | | | | | |
|-------|------|------|------|------|------|------|------|------|------|------|------|
| Years | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
| Sales | 1.3 | 1.1 | 1.2 | 1.4 | 2.1 | 2.2 | 1.6 | 1.5 | 2.0 | 2.5 | 1.6 |

Table – 2
Sales of the institute B

| Years | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
|-------|------|------|------|------|------|------|------|------|------|------|
| Sales | 1.1 | 1.2 | 1.4 | 2.1 | 2.2 | 1.6 | 1.5 | 2.0 | 2.5 | 1.6 |

- Direct the students to plot a graph for the given data in the table
- Guide them to obtain two coordinates for making the linear trend.
- Prepare them to divide the set of data into two equal parts and calculate the Means in each part. (When the Time Series consists of an odd number of values, the middle value should be ignored.)
- Advise them that the average of each part is centered in the period of time of the part from which it had been calculated.
- Plot the above two coordinates against the middle of the time span on the graph and thus draw a line to pass through the plotted points.
- Guide the students to obtain the Gradient and Intercept of the line and the equation of line as $Y = A_0 + A_1X$
- Guide them to state the advantages and disadvantages of this method relatively to Free hand method.
- Get the students prepared for a collective and constructive presentation with their findings before the entire class.

Guidelines to understand the subject matters:

- The time series is first divided into two parts and calculate the means of each part and consider each mean correspond to the centre of relevant period. The trend line can be obtained by joining these two means.
- This method is called Semi-Averages method.
- When there is an odd number of values of a time series the middle value of the series should be omitted.
- Advantages of obtaining the trend line by using Semi-Averages method are as follows:
 - Only one trend line can be obtained for a particular time series
 - Reliable decisions could be made because of the trend line is derived a using mathematical technique free from subjective elements.
- Disadvantages of obtaining the trend line by using Semi-Averages method are as follows:
- The limitations of the Mean may be included in this method, since this method based on Arithmetic Mean.
- This technique could not be sufficient for estimating the trend line because the middle value of the series that consists an odd number of values is omitted.
- Can cause misinterpretation about the trend because this method is not flexible in calculation.

Competency Level 7.5 : Calculate the trend in such a way that errors are minimized
No of Periods : 04

Learning Outcomes :

- States the steps for estimating linear trend using least square method.
- Estimates the linear trend.
- Describes relative importance of the least square method for estimating trend.
- Explains the complex situations using simple techniques.
- Makes management decisions so that errors are minimized

Learning -Teaching Process:

Approach:

- Present the following equations to the class.

$$\sum Y = n\beta_0 + \beta_1 \sum X$$
$$\sum XY = \beta_0 \sum X + \beta_1 \sum X^2$$

Enquire the class about the following expressions

$$\sum Y, \sum X, \sum XY, \sum X^2$$

- Lead a discussion highlighting the following facts
- X - independent variable, Y - dependent variable,
- β_0 - Intercept β_1 - Gradient
- Estimate the linear trend based on the above two normal equations.
- When the time is independent variable, time variable is adjusted appropriately.
- We can edit and simplify the above normal equations because of the time is a variable which contains successive values.

Proposed Instructions for learning:

- Divide the students into three groups and provide the tables given in competency level 7.1
- Guide the students to identify origin of variable X and give the relevant codes consequently for other periods.
- Advise them to quantify β_0 and β_1 substituting X and Y values for the above normal equations.
- Discuss the reasons for editing the equation with respect to the time series.
- Guide the students to quantify β_0 and β_1 by using successive values considering the point of origin on the starting year.

- Compare the values of β_0 and β_1 estimated using above two methods.
- Plot the above equations and ask the point of views of students
- Lead the groups to inquire comparatively the advantages of this method with relative to other methods of estimating trend.
- Prepare for a collective and instructive presentation with your findings before the entire class.

Guidelines to understand the subject matters:

- Least Square Method can be used to estimate the linear trend of a Time Series.
- Since time is a variable which contains successive values, normal equations are adjusted in the estimation of trend.
- Codes should be used selecting an origin for the time variable so that $\sum X = 0$
- By making codes as above, the following formulas can be used to estimate β_0 and β_1

$$\hat{\beta}_0 = \frac{\sum Y}{n}$$

$$\hat{\beta}_1 = \frac{\sum XY}{\sum X^2}$$

- Considering the point of origin as the starting period, successive values can be used.
- When the successive values are used the following formulas can be used to estimate β_0 and β_1

$$\sum Y = n\beta_0 + \beta_1 \sum X \rightarrow (1)$$

$$\sum XY = \beta_0 \sum X + \beta_1 \sum X^2 \rightarrow (2)$$

- Estimated trend line is as follows:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$$

- Advantages and disadvantages of least square method of estimating trend are as follows:
 - Since this is a mathematical method, free from subjective errors.
 - The estimated trend is more accurate relative to other two methods.
 - More reliable because of all data of the variables are used
 - The forecasting can be done easily since trend is given by a straight line.
 - More time is spent compared to the freehand method.

Competency Level 7.6 : Explains non linear trend by smoothing time series.

No of Periods : 06

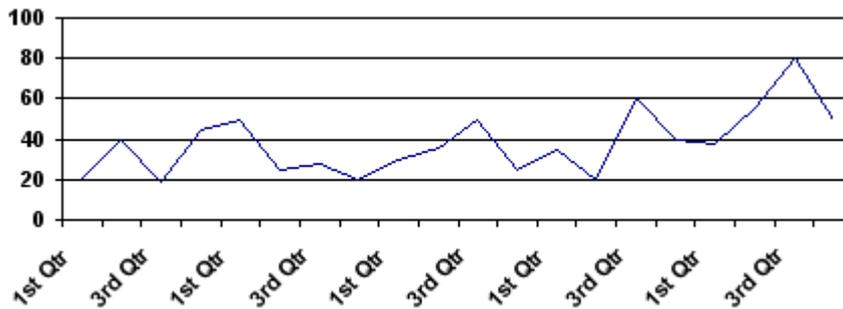
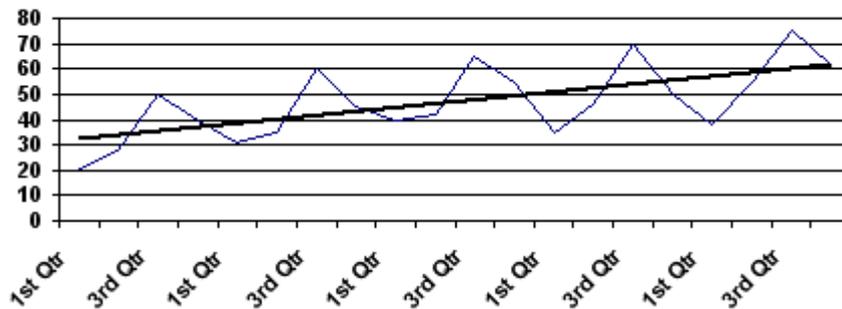
Learning Outcomes :

- States the steps for estimating trend by moving averages method.
- Calculates the trend of a time series using moving averages method
- Analyzes difference between linear trend and trend which is calculated by moving averages method
- Makes decisions by understanding the fact that it is not only linear patterns of the variables that exist.
- Responds to natural patterns of variables within difference

Learning -Teaching Process:

Approach:

- Show the two graphs given below to the class



- Give the opportunity to students to point out the differences between two graphs.
- Lead the students to select appropriate graph that is suitable to analyze the linear trend.

- Highlighting the following facts conduct the discussion
- According to the first graph annual variation has been occurred as a pattern but in the second graph it is not like as the first.
- For the first graph linear trend is suitable. But for the second graph nonlinear trend by smoothing is suitable.

Proposed Instructions for learning:

- Pay attention of two small groups to the time series shown in the table no . 2 in the competency level 7.1
- Guide first group to use moving averages of 3- period and to the second group to use 5- period moving averages.
- Display a large graph of original data on the blackboard and give chance to the two groups to calculate the period 3 and period 5 moving averages.
- Calculate the period 2 and period 4 moving averages and plot on the same graph with different columns.
- Examine the constructed four graphs and study the special features in them.
- Give instructions to the student when taking moving averages with large number of periods there are some errors.
- Using another example of monthly data construct graphs and compare the difference between first and second situation
- Give chance to the student to find the optimum time period to calculate the moving averages using their experience.
- Prepare for a collective and constructive presentation with your findings before the entire class

Guidelines to understand the subject matters:

- Degree of time period is important when smoothing for eliminating the seasonal variation
- When calculating trend using moving averages method data are lost at two ends.
- When the degree of period is large the number of lost data is also large.
- A smooth curve can be identified from moving averages method

Competency Level 7.7 : Uses simple methodology to compute the seasonal index
No: of Periods : 06

Learning Outcomes :

- Defines the seasonal index correctly
- States the steps to compute seasonal index using average percentages method
- Computes seasonal index for the given time series data
- Make decisions logically identifying the seasonal effect of variables come across in the daily life.

Learning - Teaching Process:

Approach

- Give the following two events to the class
Somapala and Gunapala were two textile traders in last April. Somapala was in very difficult stage because he could not supply enough materials for the demand. But Gunapala faced different situation that he had stocked a lot of good in Vesak month and could not sell those.
- Discuss in the class the reasons why they have faced these situations. To avoid these situations explain how these two must behave hereafter
- Conduct a discussion by highlighting these facts
- Cultural and social facts affect the businesses
- The variations which repeat in less than one year are called seasonal variations
- Because the two traders could not identify these variations in advance they suffered
- The computing of seasonal index is important to identify these situations in advance

Proposed Instructions for learning:

- Divide the class in to small groups and ask students to buildup situation with data which helps to understand the seasonal indexes.
- Distribute among the groups reading materials which includes the steps of calculating seasonal indexes.
- According to the build up data ask students to compute the seasonal indexes.
- Analyze the demerits of calculating seasonal index thinking as you are the group faced for the above situation

Guidelines to understand the subject matters:

- Give attentions for the following reading materials
- Reading materials
- I am the principal of Nisanka collage. Visitors who came to meet me during the office hours within the week of month of June had been recorded and wanted to identify the patterns of visitor's arrivals. Following are the way of how I constructed the five indexes for each day. These indexes help me in planning daily work.

| No. of visitors arrived in each day | | | | | | Arrivals during the week | Average arrivals during the week |
|-------------------------------------|----|----|----|----|----|--------------------------|----------------------------------|
| | M | T | W | Th | F | | |
| week 1 | 18 | 12 | 33 | 15 | 12 | 90 | = 18 |
| week 2 | 16 | 14 | 31 | 20 | 14 | 95 | = 19 |
| week 3 | 28 | 22 | 48 | 14 | 13 | 125 | = 25 |
| week 4 | 21 | 26 | 40 | 18 | 15 | 120 | = 24 |

The original data as a percentage of mean

| | | | | | |
|--------|--|--|---|---|--|
| week 1 | $\frac{18}{18} \times 100$ (100%) | $\frac{12}{18} \times 100$ (66.67%) | $\frac{33}{18} \times 100$ (183.33%) | $\frac{15}{18} \times 100$ (83.33%) | $\frac{12}{18} \times 100$ (66.67%) |
| week 2 | $\frac{16}{19} \times 100\%$ (84.21%) | $\frac{14}{19} \times 100\%$ (73.68%) | $\frac{31}{19} \times 100\%$ (163.16%) | $\frac{20}{19} \times 100\%$ (105.26%) | $\frac{14}{19} \times 100\%$ (73.68%) |
| week 3 | $\frac{28}{25} \times 100$ (112%) | $\frac{22}{25} \times 100$ (88%) | $\frac{48}{25} \times 100$ (192%) | $\frac{14}{25} \times 100$ (56%) | $\frac{13}{25} \times 100$ (52%) |
| week 4 | $\frac{21}{24} \times 100$ (87.5%) | $\frac{26}{24} \times 100$ (108.33%) | $\frac{40}{24} \times 100$ (166.67%) | $\frac{18}{24} \times 100$ (75.0%) | $\frac{15}{24} \times 100$ (62.5%) |

Total of

percentages 383.71 336.68 705.16 319.5 254.85

Mean 95.93 84.17 176.29 79.90 63.71

(Seasonal Index)

Competency Level 7.8 : Uses ratio to moving averages method to compute the seasonal index

No: of Periods : 06

Learning Outcomes :

- States the steps to compute the seasonal index using the ratio to moving averages method.
- Calculate seasonal index of a time series having quarterly date or monthly data using the ratio to moving averages method.
- Identifies the relation between the seasonal indexes calculated relevant to the each month or quarter and corresponding original values.
- Makes logical decisions covering unfavorable situations with the favorable situations in the business field
- Behaves comparatively with fluctuations faced in society

Learning - Teaching Process:

Approach

- Present the following dialogue to the class

Sumanaweera: How is your business these days

Chandrapala : No considerable gain in this month. Covered this month expenditure without borrowing, since there were more profit in march and April

Lead a discussion highlighting the following facts.

- Seasonal effect of a time series, can be smoothed among consecutive units of time
- By considering seasonality on equal time interval, Seasonal variations are sharply highlighted
- Moving averages balance the fluctuations of a variable.
- Seasonal index can be calculated using the moving averages method also.

Proposed Instructions for learning:

- Divide the students into two groups
- Draw the attention to the time series data with seasonal variations constructed of the competence level 7.7
- Make exploration giving following instructions
 - Order the monthly or quarterly data in each year consecutively
 - Calculate the centered moving averages using degree 4

- Express original value of each quarter or month as a percentage of corresponding moving average.
- State those results in the following table

| | |
|-----------------------|--|
| Quarter/ Month | |
| Year | |
| | |
| Total | |
| Average | |
| Seasonal Index | |

- Adjust if the total of averages of 4/12 are not 400/ 1200
 - explain deviations considering average of each index as 100
 - State the advantages and disadvantages of moving ration method with compared to average percentage method.
- Prepare for a collective and constructive presentation with your finding before the entire class

Guidelines to understand the subject matters:

- The most accurate method of computing seasonal index is ratio to moving average method comparatively
- When original data of the time series are divided by the moving average only irregular variations and seasonal variations remain. Those are called as short-term variations. S.I.
- When taking centralized moving averages S. I., are eliminated and long term variations of T.C are shown from moving averages
- By eliminating irregular variations of (I) from short term variations of S.I. , the seasonal indexes can be obtained.
- However following unfavorable effects can be seen
- Inclusion of common disadvantages of the mean in the index
- According to the size of degree of interval, some data are lost from two ends
- Some variation which are not in the original data, can be included in the index

Competency Level 7.9 : Takes correct decisions using deseasonalized data.

No: of Periods : 04

Learning Outcomes :

- Expresses the nature of deseasonalized data
- Give examples for time series which has seasonal variations
- Analyses drawing graphs of deseasonalized data and original data.
- Makes decisions removing the seasonal variation of variables.
- Uses the knowledge about pattern of seasonal variations to better utilization of resources.

Learning - Teaching Process:

Approach

- Present the following dialogue to the class calling two students in front of the class
Trader: (Owner of the shop)

Sales of cloths in December increases 200%. Therefore I made more cloths on month of January for sale. But lot of stock were remand in the stores. It was loss for me because I spent a lot of money at once.

Marketing agent

You have taken wrong decision December is a festival month. Because of festivals you had sales. But in January normally the sales of cloths will decrease without identifying these if you do the business you will get losses.

- Make a discussion highlighting the following facts
- When making decisions there should be a knowledge about the short-term fluctuations
- The discussion taken based on temporary fluctuations can be wrong
- Trend can be taken by Deseasonlzing the time series data.

Proposed Instructions for learning:

- Group the class as did in competency 7.7
- Draw the attention for the time series which students constructed with seasonal variations at that competence level
- Consider the stational indexes constructed in 7.7 and 7.8
- Divide the time series original data from the seasonal index, constructed by themselves

- Draw in one Cartesian plane the original data and data calculated from the above
- Examine students ideas about the two lines
- Prepare for a collective and constructive presentation with your findings before the entire class.

Guidelines to understand the subject matters:

- The actual nature of the time series can be identified after removing the temporary variations
- Temporary variations of the time series except irregular variations are seasonal variations
- By deflating the original data from seasonal index the seasonal variations are eliminated
- By drawing in the same Cartesian plane the deflated data and original data nature of the time series can be identified well if there is no seasonal effect

Competency Level 7.10 : Forecasting using analyzes of time series components

No: of Periods : 04

Learning Outcomes :

- Expresses the way of forecasting done by time series
- Forecast using the given data
- Analyses the validating of the forecasted data
- Takes decisions about future based on present situation
- Utilizes resources efficiently through forecasting

Learning - Teaching Process:

Approach

- Present the following broadcasting statements to the class by a student

A part of a news broadcast

- According to the last ten years records of annual arrival of tourist to Sri Lanka, it is expected that 50000 would arrive this year. The meeting held at ministry of tourism with minister revealed that the facilities which required in tourism sector are to be expanded.
- Conduct a discussion highlighting the following facts
 - Forecasting is necessary for planning
 - Past data are used for forecasting
 - Time series data can be forecasted using trend and secondly indexes

Proposed Instructions for learning:

- According to the following activities divide the students into two groups
- Using the given trend equation calculate the values for 4 quarters of your 2010
- Estimate the trend value by substituting those values.
- Forecast the data for year 2010
- List the decisions that management can make based on the forecasted values
- Results of a time series analysis conducted by ministry of tourism affairs regarding the quatterly migrations of tourists from 2005 to 2009 are stated below.

- Trend line free from seasonal movements

$$\hat{Y} = 3 + 1.2t$$

(A unit of t is half of a quarter and units of Y - are in thousands)

Seasonal indices for 4 quarters

| Q1 | Q2 | Q3 | Q4 |
|-------|-------|-------|--------|
| 98.33 | 92.25 | 108.9 | 100.52 |

- Given below are the results of a study regarding sales from 2005 to 2009 conducted by an entity whose revenues are trend to be declining

Trend line free from seasonal Movements

$$\hat{Y} = 2.3 - 0.5t$$

(A unit of t is $\frac{1}{2}$ of a quarter and units of Y in thousands)

Seasonal indices for 4 quarters

| Q1 | Q2 | Q3 | Q4 |
|----|-----|----|-----|
| 95 | 110 | 80 | 115 |

Guidelines to understand the subject matters:

- The linear time series trend equation should be constructed using the data given
- Monthly or quarterly seasonal indices should be computed related to those data.
- The trend value for given future time units can be obtained, using the seasonal indices an

long term trend in the model $\hat{Y} = \left(\hat{E}_0 + \hat{E}_1 \times \right) S / 100$

- Reliability/ Validity of such forecast is restricted only for few closer periods to the time mentioned in data. This can be regarded as a limitation of time series analysis.

Competency 8.0 : Uses the statistical techniques in making management decisions.

Competency Level 8.1 : Uses the statistical techniques to produce quality goods and services by controlling variations in qualities

No: of Periods : 08

Learning Outcomes :

- Constructs definitions for quality control by identifying basic terms
- Logically highlights the importance of quality control using goods and services as examples
- Constructs the concept maps to analyse the factors which affect the quality of a produc.
- Makes the correct selections considering the quality
- Uses the concept maps to ease the study

Learning - Teaching Process:

Approach

- Exhibits the enlarged advertisement which contains the details of few goods which have SLS/ISO, and few goods which do not have SLS/ISO and few services which have standards and few services which do not have standards
- Randomly select a student and ask the questions about the goods and services he likes to buy from the above goods and services and the reasons to select those goods & services
- Lead a discussion highlighting the following facts.
 - Consumer considers the quality of the product when buying goods & services
 - Quality of a product is a standard which are predetermined by the production institution and those standards should be included in the good or service
- Statistical quality control examines whether the goods or services are produced according to the pre determined standards and specifications.

Proposed Instructions for Learning:

- Guide the groups to collect the information from school from production or service places around the schools under the following situations
 - Quality control of goods produced
 - Quality control of services provided
- Does the supplier of the good or service concern about the quality? If so in what way?
- Do they bear a cost to control the quality? How much is it? What is the benefit received for the cost?

- Are there any controllable factors which affect the variations of quality. If so what are they?
- Give Instructions to students to construct the concept map which help to highlight the factors which affect the variations of quality of the good or service.
- Statistical quality control examines whether the goods and services are produced according to the predetermined standards and specifications.

Guidelines to understand the subject matters:

- Production organization can get the following advantages from quality control.
 - Possible errors can be identified early
 - Logical decisions have to be made by sample surveys because practically in large scale production every item cannot be examined.
 - It is more efficient to estimate standard of the population using sample taken from the population.
 - There are situations where all the units of a good cannot be examined for their quality. Therefore few items have to be examined and make decisions about the whole.

Examples: The flash bulbs cannot be used again after examination.

- Guarantees of good quality can be achieved with low cost of investigation.
- Less possibilities of rejecting the goods by the consumers.
- The highest level manager to lower level worker participate in quality control process.
- Productivity can be increased.

- Two main factors affect the variations in quality of a product
 - Random factors
 - Non random factors
- Variations due to random factors are called Random/ Chance/Allowable variations.
- Random variations are beyond human control and it will take high cost to eliminate them. Therefore these facilities are ignored.

Ex. Variation of the quality of product due to physical factors such as temperature, humidity.

- Variations, due to non random factors are called assignable / controllable/ preventable variations
 - Ex.* Depreciation of machine, Outdated machine and Untrained workers cause defective products
- After identifying the factors which cause the assignable variation, the variation due to them can be controlled.

Competency Level 8.2 : Uses proper methods to control the variations

No: of Periods : 10

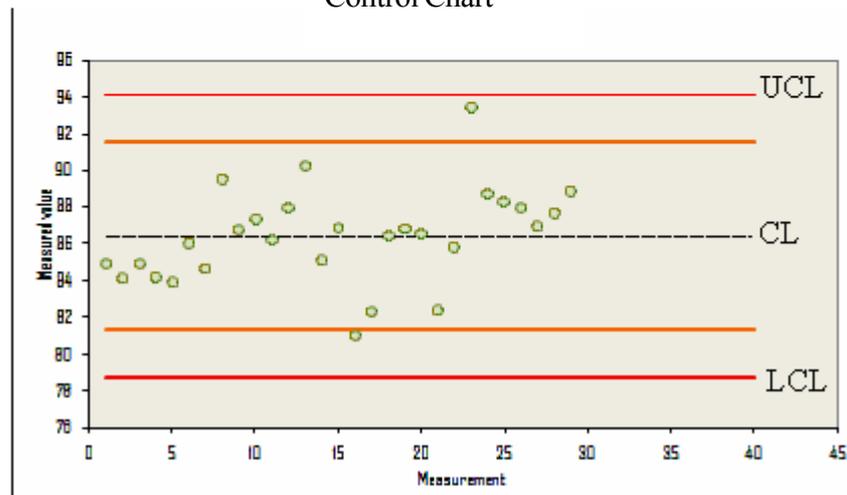
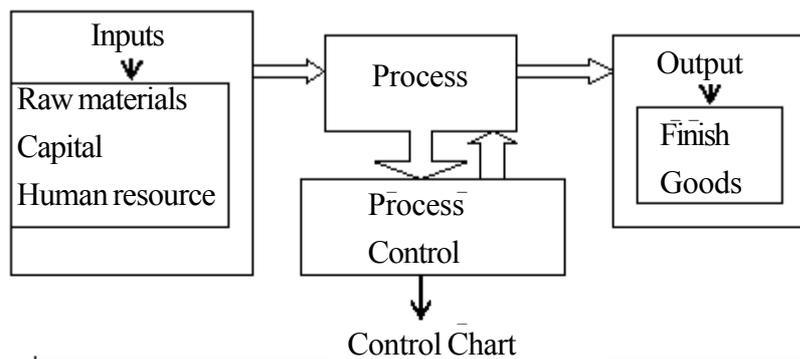
Learning Outcomes :

- States the methods of controlling variations and presents the control limits for each one.
- Constructs control charts using control limits
- Analyses effect on product by interpreting control charts
- Uses diagrams to ease the interpretation;
- Pays attention to the process from the beginning to the end for having optimal output.

Learning - Teaching Process:

Approach

- Present the following diagram to the class



- Give chance to the students to name the 3 elements relevant to the diagram.
- Examine the factors which affect the finish good
 - If one or few points are out side of the control limits what is expected by students about quality of the product
- Lead a discussion highlighting the following facts.
 - Basic elements relevant to product are inputs, process and output.
 - Control charts are used as statistical techniques which used to control the process
 - Process can be controlled by controlling variations between upper and lower limits.
 - If at least one value is out side the control limits, the process is not under control.
 - Statistical methods are used to decide the control limits under two situations.

Proposed Instructions for Learning:

- Draw students attention to the following data
- Taking samples of five packets per day from ten consecutive days from biscuit manufacturing firm and weighted each packet and reported as follows.

| Day | Weight of biscuit packets (gm) | | | | |
|-----|--------------------------------|-----|-----|-----|-----|
| | Sample Number | | | | |
| | 1 | 2 | 3 | 4 | 5 |
| 1 | 402 | 405 | 400 | 398 | 395 |
| 2 | 395 | 398 | 405 | 410 | 403 |
| 3 | 412 | 390 | 395 | 388 | 482 |
| 4 | 415 | 399 | 400 | 390 | 395 |
| 5 | 395 | 403 | 405 | 412 | 380 |
| 6 | 385 | 390 | 398 | 415 | 410 |
| 7 | 396 | 390 | 400 | 412 | 415 |
| 8 | 390 | 400 | 400 | 410 | 385 |
| 9 | 380 | 395 | 390 | 395 | 385 |
| 10 | 405 | 408 | 410 | 395 | 395 |

- Draw the attention to the situation which each group receive from the following two situations.
 - Process is under control.
 - Process is not under control.
- According to the following instructions guide students in learning
 - Examine the meaning of the process which is under control and process which is not under control
 - Give the data in the above table to the group which gets, "the process is under control."
 - Change day 9 th sample data as 370, 385, 375, 385, 370 and give to the group which gets, "the process is not under control."
 - Give instructions to calculate the Mean (\bar{X}) and Range (R) for each sample data.
 - Calculate the mean of the means $\bar{\bar{X}}$ (Grand mean) and mean of the ranges (\bar{R})
 - Give instructions to take value of $\bar{\bar{X}}$ as central line of the \bar{X} chart and value of \bar{R} as central line of the \bar{R} chart
 - Guide to calculate the upper control limit and lower control limit for each control chart using formulas

- **\bar{X} chart**
 - Upper control limit
$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{R}$$
 - Lower control limit
$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{R}$$
- **Range chart**
 - Upper control limit
$$UCL_R = D_4 \bar{R}$$
 - Lower control limit
$$LCL_R = D_3 \bar{R}$$

- Give instructions to the students to read the value relevant to factors of A_2, D_4 and D_3 for n=5 from the quality control table
- Examine whether the process is under control or not, by constructing the mean control chart.
- Examine whether the process under control or not, by constructing the range chart

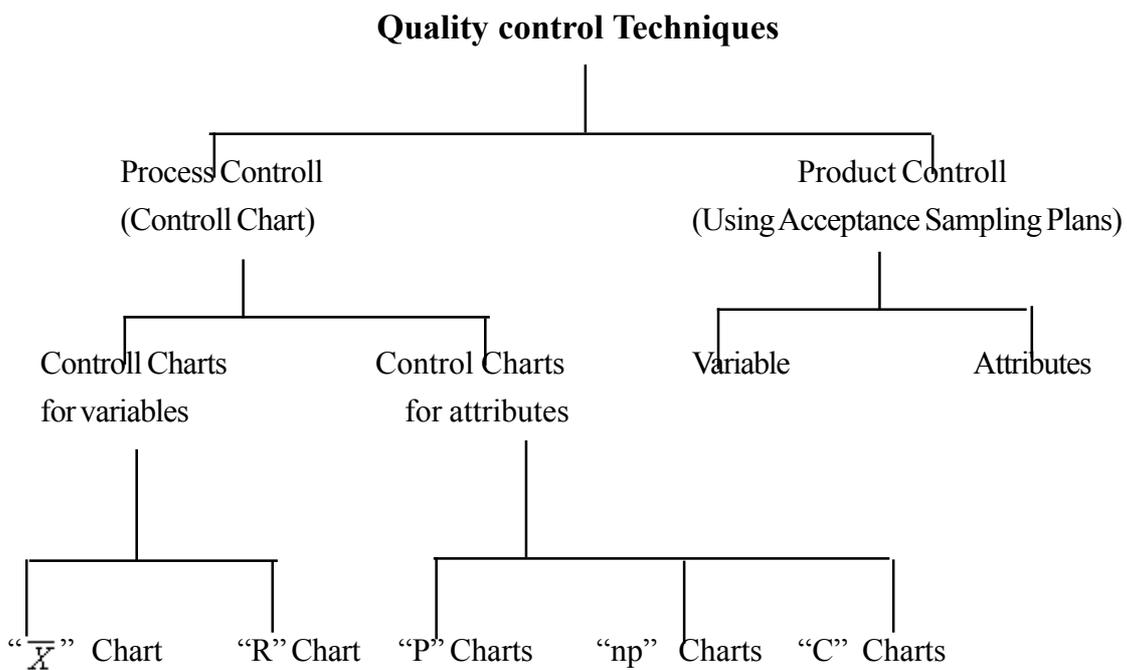
- Assuming that the firm has determined the mean weight of a packet of biscuit as 400 gm (μ) and standard deviation as 10 gm (σ), give instructions to calculate control limits again for the \bar{X} and R charts using formulas.

| |
|--|
| $\bar{x} \text{ Chart}$ $UCL_{\bar{x}} = \mu + A\sigma$ $CL_{\bar{x}} = \mu - A\sigma$ $R \text{ Chart}$ $UCL_R = D_2\sigma$ $CL_R = d_2\sigma$ $LCL_R = D_1\sigma$ |
|--|

- Give instructions to the students to check whether the calculated \bar{x} and R values are within the control limits.
- Prepare for a creative and collective presentation before the entire class with your findings

Guidelines to understand the subject matters:

- Quality control techniques can be shown by a flow chart as follows



- Examining whether the production process is behaving according to the expected levels is called process controls. Hence the variations in the quality of the good can be identified.
- Control charts which are used to control the quantitative characteristics are called as control charts for variables

Examples:

- Weights of a sugar packet
 - Length of a piece of timber
- Control chart is a chart which consist of three horizontal lines, “Central Line” which shows the required standards of the production process and two other lines which give the upper and lower control limits. (The x – axis of the graph represents the serial number of the samples and Y - axis represents the corresponding value of the qualitative characteristic in the Cartesian plain
 - When constructing the control charts it is assumed that the relevant variables are normally distributed
 - Control charts are constructed by giving chance to sample data to vary within range of 3 standard deviations ($\mu \pm 3\sigma$) from the mean.
 - When the calculated value for lower control limit is a negative value, horizontal axis (zero line) is taken as lower limit.
 - Mean control chart (\bar{x} chart) is considered under two situations.
 1. When population mean μ and standard deviation σ are known.
 2. When population mean μ and standard deviation σ are unknown.
 - Mean charts are prepared by using means of observations about the standards of the samples taken during the production process.
 - The following formulas can be used when constructing control limits of \bar{x} charts when standards are given.

1. Upper control limit

$$UCL = \mu^1 + 3\sigma_{\bar{x}}$$

Central Line

$$CL_{\bar{x}} = \mu^1$$

Lower control limit

$$LCL_{\bar{x}} = \mu^1 - 3\sigma_{\bar{x}}$$

$$\left[\sigma_{\bar{x}} = \frac{\sigma^1}{\sqrt{n}} \right]$$

The above formulas/ equations can be transformed as follows.

2. Upper control limit

$$UCL_{\bar{x}} = \mu^1 + A\sigma^1$$

Central line

$$CL_{\bar{x}} = \mu^1$$

Lower control limit

$$LCL_{\bar{x}} = \mu^1 - A\sigma^1$$

(The value of A relevant to the sample size n is expressed by the factor $\frac{3}{\sqrt{n}}$ and these values under the factor A can be taken from the quality control table.)

- When standards are not given following equations can be used to determine the control limits of \bar{x} chart.

1. Upper control limit

$$UCL_{\bar{x}} = \bar{x} + 3\sigma_{\bar{x}}$$

Central line

$$CL_{\bar{x}} = \bar{x}$$

Lower control limit

$$LCL_{\bar{x}} = \bar{x} - 3\sigma_{\bar{x}}$$

- Following equations are used for calculations.

2. Upper control limit

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2 \bar{R}$$

Central line

$$CL_{\bar{x}} = \bar{\bar{x}}$$

Lower control limit

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2 \bar{R}$$

(The values of A_2 factor relevant to the sample size n can be taken from quality control table).

- Range charts (R charts) are used to control the production process.
- The range (R) is calculated from the observations of each sample taken from the production process.
- Chart which is constructed by calculating control limits using ranges is called Range chart.
- Control charts for Range can be considered under two situations.
 1. When standards are given.
 2. When standards are not given.
- When standards are given as σ^1 for standards deviation σ .

Upper control limit

$$UCL_R = D_2 \sigma^1$$

Central line

$$CL_R = d_2 \sigma^1$$

Lower control limit

$$LCL_R = D_1 \sigma^1$$

(Value of D_1 , d_2 and D_2 factors for different size of n can be taken from quality control tables).

- The following formulas can be used to construct range charts when standards are not given.

Upper control limit

$$UCL_R = \bar{R} + 3\sigma_R$$

Central line

$$CL_R = \bar{R}$$

Lower control limit

$$LCL_R = \bar{R} - 3\sigma_R$$

- Standard deviation of ranges can be obtained from the following formula.

$$\sigma_R = \sqrt{\frac{\sum (R - \bar{R})^2}{K}}$$

[K = number of samples, if $n \leq 30$
K - 1 should be used instead of K in the above formula]

- The Mean chart and the Range chart are both important to make a optimum decision.

Competency Level 8.3 : Uses suitable methods to control the attributes.

Number of Periods : 10

Learning Outcomes :

- Explains what is controlling of attributes.
- Calculates relevant control limits for p charts, np charts and c charts using formulas / equations.
- Constructs p chart, np chart and c chart.
- Makes decisions logically.

Learning - Teaching Process :

Approach

- Present the following questions to the class.
 - When you are buying a mango from the market which characteristics in the mango would you consider?
 - When you are going to doctor to take treatment for sickness which characteristics of doctor would you consider?
- Lead a discussion by highlighting the following facts.
 - Characteristics which belongs to people or goods are called attributes.
 - Control charts can be used to control these attributes.

Proposed Instructions for Learning :

- Give following data to two groups. Direct one group for A_1 and A_2 tables and the other group for B_1 and B_2 tables.
- The following table gives the number of defective glass pieces in the 10 samples of glass pieces of sample size of 10.

| | A_1 | | | | | | | | | |
|--------------------------|-------|---|---|---|---|---|---|---|---|----|
| Sample number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Number of Defectives (c) | 3 | 4 | 2 | 2 | 1 | 4 | 2 | 3 | 1 | 3 |

- Consider the number of air bubbles in the selected 10 defective glass pieces are as defectives. The number of air bubbles in each glass piece are as follows.

A_2

| | | | | | | | | | | |
|--------------------------|---|---|---|---|---|---|---|---|---|----|
| Glass piece number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Number of Defectives (c) | 2 | 3 | 1 | 2 | 4 | 5 | 3 | 2 | 4 | 2 |

B_1 - Consider defective number of glass pieces in the sample number 4 of A_1 table as 7 and for sample number 9 of A_1 table as 0.

B_2 - Consider the value of sample number 2 as 8 and the value in sample number 8 as 0 in the A_1 table.

- Give instructions to construct the control charts by calculating control limits using given equations.

P- chart (proportion chart)

$$UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$CL_p = \bar{p}$$

$$LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\bar{p} = \frac{\text{Number of defective units}}{\text{Total number of units examined / inspected}}$$

$n = \text{sample size}$

np chart (control chart for defective units)

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$CL_{np} = n\bar{p}$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

C chart

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}}$$

$$CL = \bar{c}$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

- Give your ideas about the process according to the control chart drawn.
- What is the relationship between p chart and np chart?
- How does the c chart differ from p chart?

Guidelines to understand the subject matters:

- Control chart which graphs the proportion of defective goods in the sample which was taken from the production process is called 'p' chart or proportion chart.
- The proportion of defective can be calculated from the number of defective goods divided by the total number of items in the sample.
- Standard error of the sample proportion is as follows.

$$\sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

- The Upper and Lower control limits for p charts lie above three standard deviations from \bar{p} and below three standard deviations from \bar{p} .
- Number of defectives can be obtained by multiplying the defective proportion by the sample size n. Based on that defective number, np chart can be constructed.

- To get the control limits and the central line the relevant limits should be multiplied by the sample size n.
- Control chart which is used to control the number of defects in the product is called C chart.
- C chart is used when the sample size is large and the exact number of units can not be assigned to the sample.
Ex: Number of faults in a piece of cloth, number of words with errors in a printed page.
- When the lower control limit is negative then horizontal axis is taken as lower limits for all control charts.
- Control charts can be constructed using formulas under two situations.
 - When standards are given.
 - When standards are not given.

| | Standards are given | Standards are not given |
|----------|---|--|
| P chart | $CL_p = P'$ $UCL_p = P' + 3\sqrt{\frac{P'(1-P')}{n}}$ $LCL_p = P' - 3\sqrt{\frac{P'(1-P')}{n}}$ | $CL_p = \bar{p}$ $UCL_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})}$ $LCL_p = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})}$ |
| C chart | $CL_{np} = \lambda$ $UCL_c = \lambda + 3\sqrt{\lambda}$ $LCL_c = \lambda - 3\sqrt{\lambda}$ | $CL = \bar{c}$ $UCL_c = \bar{c} + 3\sqrt{\bar{c}}$ $LCL_c = \bar{c} - 3\sqrt{\bar{c}}$ |
| np chart | $CL_{np} = n\bar{p}'$ $UCL_{np} = n\bar{p}' + 3\sqrt{n\bar{p}'(1-\bar{p}')}$ $LCL_{np} = n\bar{p}' - 3\sqrt{n\bar{p}'(1-\bar{p}')}$ | $CL_{np} = n\bar{p}$ $UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$ $LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$ |

Competency Level 8.4 : Uses relevant methods to control the product

Number of Periods : 12

Learning Outcomes :

- Explains what the acceptance sampling plan is
- Prepares the acceptance probability table according to the acceptance sampling plan
- Constructs the OC curve relevant to the given data
- Analyses the relationship between sample size, slope of the OC curve and probability of accepting the stock

- Uses graphs to explain complex situation in simple way

Learning - Teaching Process :

Approach

- Give the following dialogue occurred between producer and wholesale buyer

Producer : Shall we determine the price for this lot of goods

Buyer : Isn't there any defective in the lot

Producer : We examine the quality while producing the goods. We have quality circle and all the goods are examined well

Buyer : But it's good to examine the lot when buying also

Producer : There are 10000 goods in this lot. Then examine all

Buyer : It will be time consuming and need much labour.
Shall we examine a sample of them

Producer : Yes I agree

Buyer : I will select 50 items from this lot and examine them. But I will buy the lot if there are 2 or less than 2 defectives only

Producer : Yes I agree. Since you are examine only part, good lot also can be refused as well as a bad lot may be taken

- Lead a discussion highlighting the following facts.
 - Even if the quality of the goods is examined during the production process, when buying them as lots, it should be examined when there is a predetermined standard'

- Examining of all the items in the stock is called complete inspection
- Complete inspection will take more time, more labour and high cost
- Sample inspection is done to examine the lot of goods

- According to the above dialogue they have used “acceptance sampling plan” to examine the lot of goods.

- The following facts are included in the “acceptance sampling plan”
 - Size of the population N
 - Size of the sample n
 - Acceptance number C

- Acceptance number is the maximum number of defective items allowed in the sample to accept the stock

Proposed Instructions for Learning :

- Distribute following situations between the two groups

Situation A

- If sample size $n=50$ the lot is accepted when the number of defective items (C) is two or less than two only. For this accepting probability table is as follows.

| n= 50 | | |
|-----------------------------|---------------------------------|---------------------------------|
| Defective proportion | Mean of the distribution | Probability of accepting |
| P | $\lambda = np$ | |
| .00 | 0 | 1.0000 |
| .01 | 0.5 | . 6065 + 3033 + 0758 = 0.9856 |
| 0.02 | 1.0 | . 3679+.3679+1839 = 0.9197 |
| .03 | 1.5 | . 2231 +.3347 + .2510 =0.8088 |
| .04 | = | |
| .06 | = | |
| .07 | = | |

Situation B

- If sample size $n=50$ the lot is accepted when the number of defective items (C) is 3 or less than 3 only. For this 5 acceptance probability table is as follows

| Defective Proportion | $n=50$ Mean of the Distribution | Provability of accepting the lot |
|----------------------|------------------------------------|--|
| P | $\lambda = np$ | |
| .00 | 0 | 1.0000 |
| .01 | 0.5 | $0.6065 + .3033 + 0.0758 + 0.006 = 0.9982$ |
| .02 | 1.0 | $.3679 + .3679 + .1839 + .0613 = 0.9810$ |
| .03 | 1.5 | $.2230 + .3347 + .2510 + .1255 = .9343$ |
| .04 | | |
| .05 | | |
| .06 | | |
| .07 | | |

- Give instructions to complete the table given to each group
- Mark the defective proportion values in x axis and probability of acceptance in y axis in a cartesian plain
- Plot the acceptance probability values against the defective proportion values on the cartesian plain and draw the curve.
- Give instructions to name the curve as operational characteristic curve.
- Mark the point where probability of acceptance is 0.95 on the OC curve. Draw a line to X axis from this point and denote the defective proportion value as P_1 . Name that value as acceptance quality level (AQL).

- The area corresponding probabilities from 0.95 to 1.00 is called producer's risk (α)
- Explain ideas related to above probability
- Mark the point of acceptance probability of 0.10 on the curve. Name the value of X corresponding to this point as P_2 .
- Introduce this value as Lot Tolerance Percent Defective. The area that consists of acceptance probabilities from 0.00 to 0.10 is called consumer's risk (β).
- Explain ideas related to this probability
- Prepare a probability table with above defective proportions when the sample size is 100.
- Draw the operational characteristic curve relevant to $n=100$ on the same cartesian plain drawn for $n=50$.
- Show the difference between OC curves of $n=50$ and $n=100$.
- Explain the shape of OC curve when n is large.
- When n is constant and acceptance number c increases what will be the shape of the OC curve.

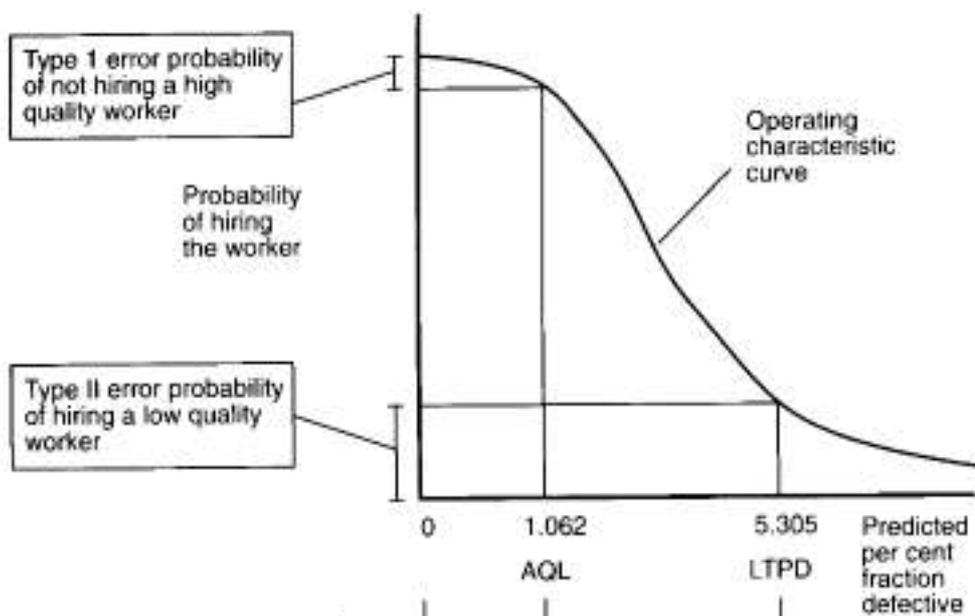
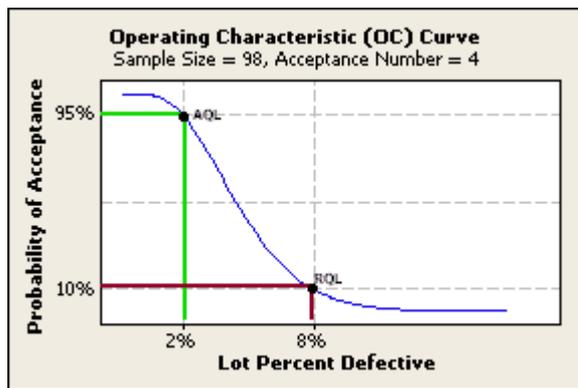
Guidelines to understand the subject matters:

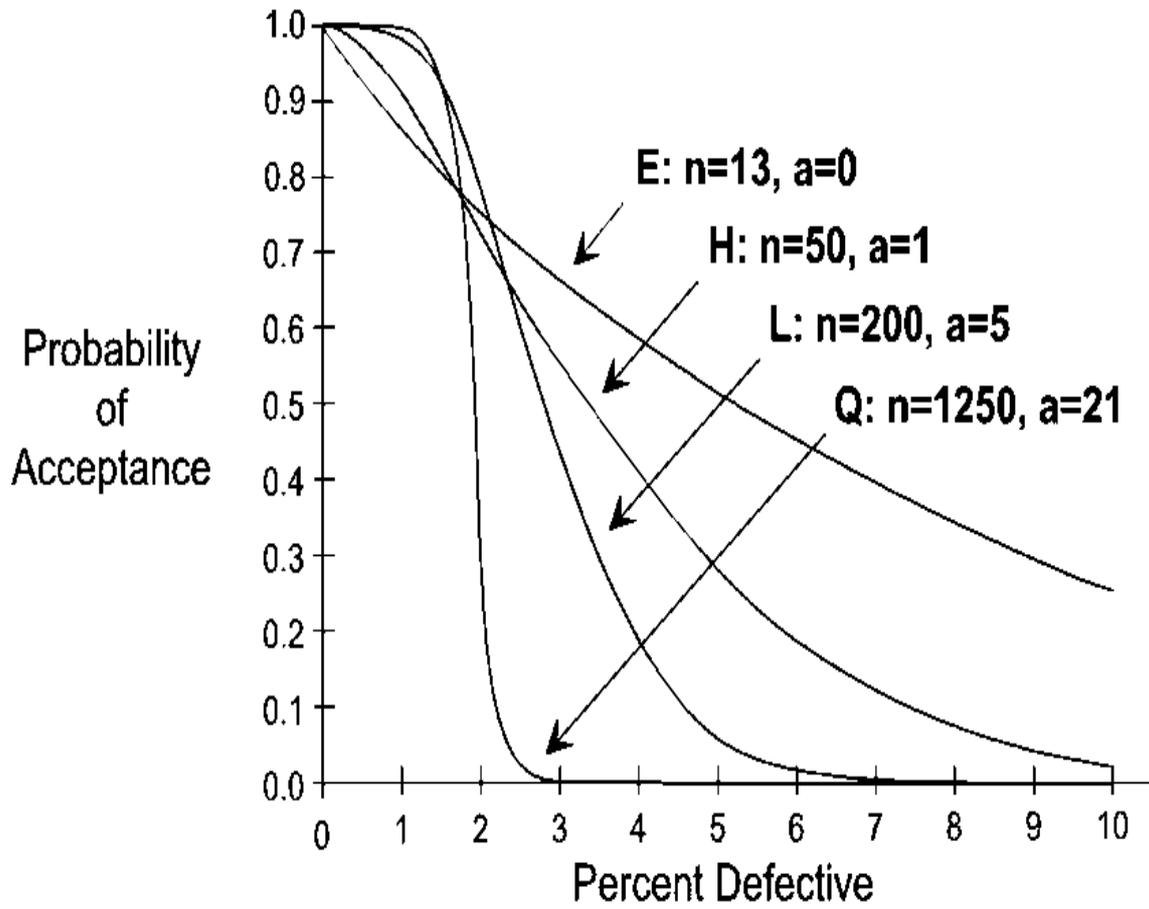
- The proportion of defectives has a Binomial distribution
- Since n is large and defective proportion is small Poisson approximation is used to calculate the acceptance probability.
- The acceptance probability of the lot corresponding to the proportion can be shown by a graph.
- That graph is named as operational characteristic curve (OC).
- By drawing OC curve following properties about acceptance sampling plan can be identified.
 - Acceptance Quality Level (AQL).
 - Lot Tolerance Percent of Defective (LTPD).
- The proportion of defectives that can be expected from an accepted lot is called Acceptance Quality Level.
- Acceptance quality level is usually maintained at 0.95 level of acceptance probability.
- Lot tolerance percent defective proportion means that the defective proportion of the bad lot.
- Lot tolerance percent defective is usually maintained at 0.10 level of acceptance probability.
- 95% of lot of goods with the proportion of defective of the acceptance quality level or less than of it is accepted.
- 10% of lot of goods with the proportion of defective at the level of lot tolerance percent defective or more than of it is accepted.
- The OC curve is used to identify the difference between acceptable lots and non acceptable lots.
- The OC curve is useful to analyze the success of acceptance sampling plan.

- The decisions that can be achieved from an acceptance sampling plan are as follows.

| | | |
|-------------------|--|--|
| Accepting the lot | Good Lot | Bad Lot |
| | Correct decision ($1 - \alpha$) | Type II error Consumer's risk (β) |
| Rejecting the lot | Type I error producer's risk (α) | Correct decision $1 - \beta$ |

- Models of OC curve are as follows.





Competency 9.0 : Behaves as a decision maker with regard to relative changes.
Competency Level 9.1 : Constructs the basic foundation to study the index numbers.
Number of Periods : 04

Learning Outcomes :-

- Identifies the basic terms of constructing the definitions.
- Explains the uses of index numbers for economic decision making.
- Tends to construct the index numbers useful to different purposes.
- Explains the changes in the present situation relatively to the base period.

Teaching - Learning process :

Approach :

- Present to the class a copy from Central Bank report which includes index numbers.
- Give an opportunity to listen to the news brief about index numbers.
- Highlighting following facts lead a discussion.
 - Index numbers are constructed for certain time intervals.
 - Use a suitable base time period to calculate the current year index numbers.
 - Express the current year value as a percentage of the base year value.
- An index number is a percentage used to compare the relative changes of a variable with respect to time, space or other characteristic.
- The value of a index number is expressed relative to the base year value.

Proposed Instructions for Learning :-

- Give the following two tables to two groups of students.

Table 1
Janaka Ltd. Company

| Time | Price of a share |
|---------------|------------------|
| January 2009 | 100 |
| February 2009 | 120 |
| March 2009 | 130 |

Table 2
Janaka Ltd. Company

| Time | Quantity purchases of good |
|---------------|----------------------------|
| January 2009 | 800 |
| February 2009 | 1000 |
| March 2009 | 900 |

- Calculate the relative changes in February and March with respect to the month of January 2009.
- Calculate the relative changes in March with respect to the month of February.
- Express the above changes as percentages.
- Give the name of the month taken as the base month.
- Which month is most suitable as base period, give reasons.

Guidelines to understand the subject matters:

- Comparison can be made in better way when the changes are calculated as percentages.
- When selecting a base period selecting a period with low fluctuations of the relevant variable is more suitable for comparison.
- Base year should not be a year which is far away from the current year.
- An index number expresses a relative change of a variable.
- It is easy to compare the changes of several variables by using index numbers.

- Index numbers are conducted to measure the changes with respect to time, space or other categories in the following variables.
 - Cost of Living.
 - Wages.
 - Import expenditure & Export income.
 - Wholesale price.
 - Retail price.

- Index numbers are used in planning, measuring of Living Standards, preparing economic policies, wage determination, etc...

Competency Level 9.2 : Makes sound decisions using simple relative indexes.

Number of Periods : 04

Learning Outcomes:

- Describes the simple relative indexes
- Explains the price relative, quantity relative and value relative.
- States the uses of simple relative indexes in making decisions.
- Uses the suitable simple relative indexes for relevant situations.
- Makes conclusions comparatively.

Learning - Teaching Process :

Approach :

- Present a price list extracted from a daily newspaper to the class as the following one.

**Colombo Maning Market
Retail Price (1 Kgm)**

| | Today price Rs: | Yesterday price Rs: |
|--------------------|----------------------------|--------------------------------|
| Samba Rice | 70 | 70 |
| Sugar | 85 | 75 |
| Dhal | 200 | 180 |
| Fish: | | |
| Tuna | 400 | 380 |
| Para | 400 | 400 |
| Yellow Fin Tuna | 460 | 440 |
| Vegetables: | | |
| Carrots | 80 | 60 |
| Beans | 60 | 50 |
| Leaks | 60 | 70 |

- Lead a decision highlighting the following facts.
 - There is a change in the current price of each good compared to the previous day price.
 - Price relatives can be used to identify the changes in the price.
 - The changes in the prices or in the quantities of any good can be represented by simple relative indices.

Proposed Instructions for Learning :-

- The prices of rice and the quantity consumed in a household for two years in the city of Colombo is given below.

| Year | Price Rs: | Quantity Kg: |
|-------------|------------------|---------------------|
| 2000 | 30.00 | 25 |
| 2009 | 70.00 | 35 |

- Using data in the above table direct the three groups to do the following activities to calculate the relatives of 1. Price, 2. Quantity, 3. Value.
 - Divide the data of 2009 by the data of year 2000.
 - Express this value as a percentage.
 - Name what does it represent.
- Taking price as P_0 , quantity as Q_0 and value as V_0 for 2000 and price as P_n quantity as Q_n , value as V_n for 2009 direct the students to express relatives as a formula.

Guidelines to understand the subject matters:

- The indexes which is used to compare the price or identity or value of a good is simple relative indexes.
- Simple relative price index shows the relative changes in the prices simple relative, quantity indexes shows the relative changes in the quantities and simple relative value indexes shows the relative value in the value.
- When constructing indexes the base year is denoted by “O” and current year is denoted by “P” quantity is denoted by “Q” and value is denoted by “V”
- Considering only one good and not considering the relative importance of goods are the disadvantages of these indexes.
- The simple price relatives have following properties
 - Identity property
 - Cyclical or Circular property
 - Factor reversal property
- Generally % sign is not used with index number
- Using index numbers the variations related to business selector can be identified.

Reading Materials

- Characteristics of simple price relations

1. Identify property

Price relative for base year relative to the same base year is equal to 1 or 100%

Ex: Price and quantities of

| | Samba rice | |
|------|------------|----|
| | P | Q |
| 2003 | 50 | 10 |
| 2006 | 60 | 8 |
| 2009 | 70 | 7 |

Answer

2003 = 50 2006= 60 2009= 70

$$\text{Base year} = \frac{50}{50} = 1 \quad \text{or} \quad = \frac{50}{50} \times 100 = 100$$

This means that the base year index is considered as 100

2. Time- Reversal Property

If the time periods of two price relations are changed each other the resulting price relations are equals to the reciprocals of price realations

Ex

| | P | Q |
|------|----|----|
| 2003 | 50 | 10 |
| 2006 | 60 | 8 |
| 2009 | 70 | 7 |

Answer

$$\begin{array}{l} 2003 \quad \text{and} \quad 2006 \\ 2003 = \frac{50}{60}, \quad 2006 = \frac{60}{50} \\ \frac{50}{60} \times \frac{60}{50} = 1 \quad \frac{50}{60} = \frac{1}{\frac{60}{50}} \end{array}$$

3. Cyclic or circular property

When considering all the above years

$$\frac{50}{60} \times \frac{60}{70} \times \frac{70}{50} = 1$$

or when there is a series of prices

$$\frac{50}{60} \times \frac{60}{70} \times \frac{70}{50} = 1 \text{ etc.}$$

Factor Reversal Property

If the price relative is multiplied by quantity relative the product is the value relative

$$\left(\frac{P_n}{P_o} \right) \times \left(\frac{Q_n}{Q_o} \right) = \frac{P_n Q_n}{P_o Q_o}$$

Competency Level 9.3 : Makes sound decisions using simple relative indexes.

Number of Periods : 04

Learning Outcomes:

- Names the simple aggregate indexes
- Describes the errors of measuring the changes of price, quantity and value of group of goods
- Calculates the simple aggregate indexes

Learning - Teaching Process :

Approach :

- Demonstrate a table similar to the following which shows prices of some goods for two years.

| Good | Quantity | 2008 Price Rs. | 2009 Price Rs. |
|--------------|-----------------|-------------------------------|-------------------------------|
| Samba Rice | 1kg | 70.00 | 80.00 |
| Flour | 1kg | 80.00 | 100.00 |
| Green gramme | 1kg | 150.00 | 160.00 |
| Coconut Oil | 1L | 200.00 | 300.00 |
| Coconut | 01 | 25.00 | 30.00 |

- Call a student before the class
- Guide him to take the average of the prices for two years
- Draw the attention of students to the question that how many times the total of 2009 prices compound the total of 2008 price
- Calculate the percentages of current your average price relative to the average price of 2008
- Lead a discussion highlighting the following facts
- Indexes which are calculated using average of prices of basket of goods are called aggregate price indexes.
- Simple aggregate price index is a measure of price changes
- Simple aggregate indexes can be calculated quantities and values as well

Proposed Instructions for Learning:

- Information about four goods sold in a retail shops in a village is given in the following table.

| Year 2008 | | | | 2009 | | |
|-----------|-------------|-------------|----------|--------------|-------------|----------|
| Good | Price of Rs | Quantity kg | Value Rs | Price of 1kg | Quantity kg | Value Rs |
| Rice | 60 | 300 | | 70 | | 350 |
| Flour | 70 | 200 | | 90 | | 160 |
| Sugar | 70 | 100 | | 80 | | 120 |
| Dhal | 150 | 40 | | 200 | | 50 |

- Direct three groups of students to calculate the indexes for (i) Price (2) Quantity (3) Value using the data in the above table
- Obtain the total of price, quantity or value columns which is relevant to the group
- Consider 2008 as base year and 2009 as current year.
- Construct the simple aggregate indexes
- According to the definition of simple aggregate indexes there should be average values in the denominator and in the numerator. But the number of goods of the average in the denominator is equal to the number of goods of the average in the numerator. Therefore the index becomes a relative of totals.
- Taking the base year price as P_o , quantity as Q_o , value as V_o and current year price as P_n , quantity as Q_n , value as V_n construct a formulas for simple aggregate price index.

Guidelines to understand the subject matters:

- Three indexes can be calculated as simple aggregate indexes such as,
 - Simple aggregate price.
 - Simple aggregate quantity.
 - Simple aggregate value.
- Simple aggregate indexes are measures with the relative changes in prices, relative changes in quantities and relative changes in values.
- Not considering the relative importance of the goods and the changes in the measuring units are the disadvantages of these indexes.

Competency Level 9.4 : Makes sound decisions using the averages of simple relatives.

No: of Periods : 04

Learning Outcomes:

- Defines the average of simple relative indexes considering the basket of goods.
- Constructs the average indexes of the price, quantity and value relatives.
- Analyses the errors and the special characteristics of the calculated indexes.
- Uses the relevant measures to compare the different characteristics.
- Uses the averages to analyses the over all changes.

Learning Teaching Process :

Approach

- Present the following list prices to the class.

| | Good | Price (Rs:) |
|-----|----------------|--------------------|
| 1kg | of rice | 80.00 |
| 1m | of cloth | 100.00 |
| 1L | of coconut oil | 350.00 |
| 1 | coconut | 30.00 |
| 1 | loaf of bread | 32.00 |

- Lead a decision highlighting the following facts.
 - There are different measuring units in different goods.
 - When constructing indexes using basket of goods there is a effect of the measuring units.
 - It is more suitable to use a index which eliminates the effect of measuring units to compare changes.

Proposed Instructions for Learning :

- Give the following two situations to the groups of students.

| Item | 2008 | | | 2010 | | |
|------------|----------------|-------------------|--------------------------|----------------|-------------------|--------------------------|
| | Price P_o | Quantity Q_o | Value $P_o Q_o (V_o)$ | Price P_n | Quantity Q_n | Value $P_n Q_n (V_n)$ |
| Rice | 70 | 10 | | 85 | 12 | |
| Bread 450g | 35 | 5 | | 32 | 4 | |
| Egg 1 | 10 | 8 | | 15 | 10 | |
| Coconut 1 | 25 | 5 | | 32 | 3 | |

| | | | |
|---------|------------------------------|------------------------------|--------------------------------------|
| | $\frac{P_n}{P_o} \times 100$ | $\frac{Q_n}{Q_o} \times 100$ | $\frac{P_n Q_n}{P_o Q_o} \times 100$ |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| Total | ===== | ===== | ===== |
| Average | | | |

| Item | 2008 | | | 2010 | | |
|----------------------|----------------|-------------------|--------------------------|----------------|-------------------|--------------------------|
| | Price P_o | Quantity Q_o | Value $P_o Q_o (V_o)$ | Price P_n | Quantity Q_n | Value $P_n Q_n (V_n)$ |
| 1L of coconut oil | 200 | 1 | | 350 | 1 | |
| 1m of cloth | 125 | 3 | | 125 | 4 | |
| 225g margarine | 40 | 2 | | 48 | 2 | |
| 1Unit of electricity | 8 | 70 | | 10 | 85 | |

| | $\frac{P_n}{P_o} \times 100$ | $\frac{Q_n}{Q_o} \times 100$ | $\frac{V_n}{V_o} \times 100$ |
|---------|------------------------------|------------------------------|------------------------------|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| Total | ===== | ===== | ===== |
| Average | ===== | ===== | ===== |
| | | | |

- Give chance to the students to discover the followings.
 - Fill in the blanks in the given table to you.
 - Name the calculated indexes according to the averages you get.
 - Explains the changes in year 2010 in the price, quantity, and value of the basket of goods with compared to the year 2008
 - Explains the special features in this index with compared to the simple aggregate indexes.
 - What are the special disadvantages of these indexes.
 - Motivate the students to present creative group work.

Guidelines to understand the subject matters:

- The following steps should be followed to calculate the simple relative index.
 - Calculate the simple relatives for each good.
 - Take sum /total of these relatives.
 - Calculate the average dividing the total from the number of goods in the basket.

- The arithmetical Mean, Median, Geometric Mean and Harmonic Mean can be used to calculate the average.

- Considering practical simplicity arithmetic mean can be used to calculate the average of relatives.

- Formulas which can be used to calculate the simple relatives are shown as follows.

Simple average index of price relatives

$$\bar{P}_n = \frac{\sum \frac{P_n}{P_o} \times 100}{n}$$

Simple average index of quantity relatives

$$\bar{P}_n = \frac{\sum \frac{Q_n}{Q_o} \times 100}{n}$$

Simple average index of value relatives

$$\bar{P}_n = \frac{\sum \frac{P_n Q_n}{P_o Q_o} \times 100}{n}$$

- Relatives are independent from measuring units.
- Relatives importance of goods is not reflected in relatives.

Competency Level 9.5 : Matches correct decisions using weighted aggregate indexes.
No of periods : 10

Learning Outcomes :

- Names the indexes which consider the relative importance of goods.
- Examines the suitability of use of weighted aggregate indexes.
- Reaches the logical decisions by considering the relative importance of goods.
- Exhibits the preparation to select most suitable measure considering weighted measures.

Learning - Teaching Process:-

Approach:-

- Present the following statement to the class.

“How to measure the average changes in the prices using the index which constructed with the price mixture of goods which express with different units such as Kg, Meters, Liters.”

- Lead a discussion highlighting the following facts.
 - There are different measuring units in goods which bought for domestic consumption.
 - Difficult to explain the aggregate consumption level by studying cost of goods and services separately.
 - Therefore to study the changes in the consumption cost, considering relevant importance of goods, suitable indexes which are independent from measuring units can be used.
 - These are called as weighted aggregate indexes.

Proposed Instructions for Learning:-

- Distribute the following tables, which show the goods and prices of items, among the groups of students.

| Item | 2000 | | 2005 | | 2010 | |
|--------------------|----------------|-------------------|----------------|-------------------|----------------|-------------------|
| | Price p_o | Quantity q_o | Price p_t | Quantity q_t | Price p_n | Quantity q_n |
| Rice 1kg | 60 | 10 | 65 | 10 | 80 | 10 |
| Dhal 1kg | 60 | 2 | 100 | 2 | 50 | 3 |
| Bread 1 | 20 | 5 | 30 | 5 | 35 | 4 |
| Coconut 1 | 15 | 10 | 20 | 9 | 33 | 8 |
| Milk powder 1kg | 300 | 1 | 350 | 1 | 500 | 1 |

| Item | 2000 | | 2005 | | 2010 | |
|---------------|----------------|-------------------|----------------|-------------------|----------------|-------------------|
| | Price p_o | Quantity q_o | Price p_t | Quantity q_t | Price p_n | Quantity q_n |
| Cheese 100g | 80 | 2 | 100 | 2 | 180 | 3 |
| Butter 100g | 60 | 4 | 100 | 5 | 150 | 6 |
| Yoghurt 25g | 10 | 5 | 15 | 8 | 20 | 10 |
| Curd 30ml | 15 | 3 | 25 | 4 | 30 | 5 |
| Pastured milk | 40 | 2 | 100 | 3 | 125 | 4 |

- Give chance to the students to calculate the indexes and to test the appropriateness of them.
- Calculate the following using the data given to you in the table.
 - Laspeyers's price and quantity indexes.
 - Paache's price and quantity indexes.
 - Fisher's Ideal price and quantity indexes.
 - Typical Time price and quantity indexes.

- Analyse highlighting the special features of the each index.
- Show the situations where these indexes are used in the field of business and economics.
- Formulas which can be used in calculating Index numbers.

- Laspeyers's Price Index

$$Lp_{n/o} = \frac{\sum p_n q_o}{\sum p_o q_o}$$

- Laspeyers's Quantity Index

$$LQ_{n/o} = \frac{\sum q_n p_o}{\sum q_o p_o}$$

- Paache's Price Index

$$Pp_{n/o} = \frac{\sum p_n q_n}{\sum p_o q_n}$$

- Paache's Quantity Index

$$PQ_{n/o} = \frac{\sum q_n p_n}{\sum q_o p_n}$$

- Fisher's Price Index

$$Fp_{n/o} = \sqrt{\left(\frac{\sum p_n q_o}{\sum p_o q_o}\right) \left(\frac{\sum p_n q_n}{\sum p_o q_n}\right)} = \sqrt{Lp_{n/o} Pp_{n/o}}$$

- Fisher's Quantity Index

$$FQ_{n/o} = \sqrt{\left(\frac{\sum q_n p_o}{\sum q_o p_o}\right) \left(\frac{\sum q_n p_n}{\sum q_o p_n}\right)} = \sqrt{Lq_{n/o} Pq_{n/o}}$$

- Typical Time Price Index

$$TP_{\%} = \frac{\sum P_n q_t}{\sum P_o q_t}$$

- Typical Time Quantity Index

$$TQ_{\%} = \frac{\sum q_n p_t}{\sum q_o p_t}$$

Guidelines to understand the subject matters:

- The following indexes can be noted as weighted aggregate indexes.
 - Laspeyers's index number.
 - Parche's index number.
 - Fisher's ideal index number.
 - Typical time index.
- When calculating Laspeyers's price index, weighted from base year quantity (q_o) and for quantity index weighted from base year price (p_o).
- Therefore Laspeyers's index has upward bias.
- When calculating Paache's price index, weighted from current year quantity (q_n) and for quantity index weighted from current year price (p_n).
- Therefore Paache's index has downward bias.
- Fisher ideal index is a Geometric mean of Laspeyers's and Paache's indexes.
- Only the index that satisfies both time reversal and factor reversal qualities is fisher's index
- But the Fisher index does not satisfy the circular quality.
- When calculating the Typical time price index, weighted from typical year quantity of goods (q_t) and when calculating the typical time quantity index, weighted from typical year price (p_t).

- Average of several consecutive year is taken as typical year.
- When the weights are not reflected properly from base and current year prices and quantities, the use of typical time index is appropriate.
- For computing Colombo consumer price index the Laspeyers Method is used and to compute the Greater Colombo price index the Paache's Method is used.
- There is no difference between weighted aggregate indexes and weighted average indexes.

SCHOOL BASED ASSESSMENT

Introduction- School Based Assessment

Learning –Teaching and Evaluation are three major components of the process of Education. It is a fact that teachers should know that evaluation is used to assess the progress of learning – teaching process. Moreover, teachers should know that these components influence mutually and develop each other .According to formative assessment (continuous assessment) fundamentals; it should be done while teaching or it is an ongoing process. Formative assessment can be done at the beginning, in the middle, at the end and at any instance of the learning teaching process.

Teachers who expect to assess the progress of learning of the students should use an organized plan. School Based Assessment (SBA) process is not a mere examination method or a testing method. This programme is known as the method of intervening to develop learning in students and teaching of teachers. Furthermore, this process can be used to maximize the student’s capacities by identifying their strengths and weaknesses closely.

When implementing SBA programmes, students are directed to exploratory process through Learning Teaching activities and it is expected that teachers should be with the students facilitating, directing and observing the task they are engaged in.

At this juncture students should be assessed continuously and the teacher should confirm whether the skills of the students get developed up to expected level by assessing continuously. Learning teaching process should not only provide proper experiences to the students but also check whether the students have acquired them properly. For this, to happen proper guiding should be given.

Teachers who are engaged in evaluation (assessment) would be able to supply guidance in two ways. They are commonly known as feed-back and feed- forward. Teacher’s role should be providing Feedback to avoid learning difficulties when the students’ weaknesses and inabilities are revealed and provide feed-forward when the abilities and the strengths are identified, to develop such strong skills of the students.

Student should be able to identify what objectives have achieved to which level, leads to Success of the Learning Teaching process. Teachers are expected to judge the competency levels students have reached through evaluation and they should communicate information about student progress to parents and other relevant sectors. The best method that can be used to assess is the SBA that provides the opportunity to assess student continuously.

Teachers who have got the above objective in mind will use effective learning, Teaching, evaluation methods to make the Teaching process and learning process effective. Following are the types of evaluation tools student and, teachers can use. These types were introduced to teachers by the Department of Examination and National Institute of Education with the new reforms. Therefore, we expect that the teachers in the system know about them well

Types of assessment tools:

- | | |
|------------------------------|--------------------------|
| 01. Assignments | 02. Projects |
| 03. Survey | 04. Exploration |
| 05. Observation | 06. Exhibitions |
| 07. Field trips | 08. Short written |
| 09. Structured essays | 10. Open book test |
| 11. Creative activities | 12. Listening Tests |
| 13. Practical work | 14. Speech |
| 15. Self creation | 16. Group work |
| 17. Concept maps | 18. Double entry journal |
| 19. Wall papers | 20. Quizzes |
| 21. Question and answer book | 22. Debates |
| 23. Panel discussions | 24. Seminars |
| 25. Impromptus speeches | 26. Role-plays |

Teachers are not expected to use above mentioned activities for all the units and for all the subjects. Teachers should be able to pick and choose the suitable type for the relevant units and for the relevant subjects to assess the progress of the students appropriately. The types of assessment tools are mentioned in Teacher's Instructional Manuals.

If the teachers try to avoid administering the relevant assessment tools in their classes there will be lapses in exhibiting the growth of academic abilities, affective factors and psycho- motor skills in the students

Learning-Teaching-Assessing Plan

1. Assessing Stage : The first term. Plan 1
2. Competency Levels covered : 6.1, 6.2, 6.3
3. Subject Matters Related to the Assessment : • Population and sample.
• Parameters and statistics.
• Simple Random sampling.
• Strata Random sampling.
• Cluster sampling.
• Systematic sampling.
• Quota sampling.
• Judgement sampling.
• Convenient sampling.
• Possible errors in sampling.
4. Type of the plan : Explorations.
5. Objectives of the plan : • Choosing the required samples in connection with a given problem.
• Examining the relative eligibility of each sampling technique.
• Explaining the possible errors in sampling.
• Stating the appropriate statistics to estimate the population parameters.
6. Instructions to implement the plan,
To the teacher : • Introduce the Learning-teaching-assessing plan to the beginning of the competency level 6.1.

• Separate the class in to few groups. Guide to as sign the leadership.

• Explain the way in which the students would be undergone to assessment through the instructions mentioned below.

- Let the students know about the periods during which they could get constructive advice in the process learning.
- Inform the students that the report of exploration should be handed over to the teacher on the due date in the first week after completion of the competency level 6.3.

To students

- Engage in the activity following the teacher in instructions given below.
- Monthly progress of the retail outlets in a divisional secretariat division should have been assessed. As a pre-preparation for this task a list of 550 retail outlets has been prepared and that is to be considered as the sampling frame. An appropriate sample for the above study should have been drawn.
- Select a sample of size 35 from the above population based on the simple random sampling technique. Use the lottery system/ random number table in this task. List out the sample numbers selected.
- The business outlets in the above population have been categorized as follows.

| | |
|----------------------|-------------------|
| Large scale outlets | 110 |
| Medium scale outlets | 220 |
| Small scale outlets | <u>220</u> |
| | <u><u>550</u></u> |

- Select a strata random sample with size 35 using the proportionate allocation method.

- If there are 12 grama niladhari divisions in the Divisional Secretariat division write down the steps to be followed in selecting a two stage cluster sample considering a grama niladhari division as a cluster.
- Select a sample of 25 business outlets stating the steps of systematic Sampling Method.
- If a non random sampling is to be selected from this population what is the most appropriate non random sampling. In this case, state giving reasons.
- Relatively analyze the eligibility of each sampling technique you applied above.
- State the statistics that could be computed using the sample data and the parameters to be estimated respectively and express to which extent do they match.

Marking Scheme

| Assessment Criteria | Assigning the Marks | | | |
|---|---------------------|---|---|---|
| | 1 | 2 | 3 | 4 |
| 1. Participation for the first discussion and clarification of the objectives. 2. Selecting the sample using each technique. 3. Relative comparison of the eligibility in each sampling technique. 4. Accurate naming of sampling errors, statistics and parameters. 5. Creativity, cooperation and time management in presentation of the outcomes of the exploration. | | | | |

Learning-Teaching-Assessing Plan

1. Assessing stage : The first term. Plan 3.
2. Competency levels to be covered : 6.8, 6.9, 6.10
3. Subject matters related to the plan :
 - Estimator estimate.
 - Point estimation.
 - Qualities of a good point estimator.
 - Interval estimation.
 - Confidence intervals for population mean - μ
 - For normal distributions whose population variance is known.
 - For normal populations whose variance is unknown.
 - Small samples.
 - Large samples.
 - Non-normal populations, whose population variance is known.
 - Confidence intervals for the difference between the means of two populations ($\mu_1 - \mu_2$)
 - For the difference between the means of two normal populations whose variance is known and for difference between the means of two non-normal populations whose variance is unknown.
 - For the difference between the means of two normal populations whose variance is unknown.
 - For the difference between the means of two normal populations whose variances are unknown but known to be equal.
4. Nature of the plan : • Open book reference.
5. Objectives of the plan :
 - Understanding the problem given.
 - Reference to prescribed books in order to derive the formulas and steps required for problem solving.

- Deriving the point/ interval estimators appropriately needed for estimating the parameters.
- Achieving at conclusions using the derived outcomes.

6. Instructions to implement the plan,

To the teacher

- Inform the students that an open reference test would be done after completion of the competency level 6.10 at the beginning of the competency level 6.8
- Let the students know about the assessing criteria.
- At the end of the competency level 6.10 provide with few problems prepared by yourself to the students.
- Guide to refer to the books or notes prescribed by the teacher when necessary.
- Inform that the booklet prepared solving the problems should be handed over to the teacher during the first week after completion of the competency level 6.10

To students

- Study the problems given by teacher carefully and have a sound awareness of them.
- Derive the relevant formulas with reference to the prescribed books and notes by the teacher when necessary.
- Estimate the parameters by solving problems stating the assumptions you have made when necessary.
- Hand over the booklet you have prepared containing the solutions for the problem and conclusions achieved to the teacher on the due date.

Problems to be Presented to Students

1. When a pole known as 1m in length is measured several times in millimeters using measuring instruments and the outcomes received are mentioned below.

999, 1000, 999, 1002, 1001, 1000, 1002, 1001

- (I) Compute unbiased estimates for mean and variance of possible errors in measuring 1m length using these measuring instruments.

2. An entity that produces Jam declares that the mass of a bottle of Jam lies on a normal distribution whose mean and standard deviation are 150g and 5g respectively. Later, the mass of a bottle of Jam is changed keeping the standard deviation remained unchanged. The manager interested in the process of size 25 and calculated the mean of a bottle of Jam as 198.5g.

Compute the confident intervals for the average mass of a newly bottled Jam (μ) at .

(I) 95%

(II) 99%

(III) Explain how to achieve at a conclusion about the population mean (μ) using the confident interval you have computed.

3. An entity that produces steel bars is in need of achieving at a conclusion about the diameter of them. A sample of 100 steel bars was drawn and measured the diameter to the nearest 0.01cm, following results have obtained.

$$\sum x = 299.8 \quad \sum x^2 = 899.15$$

(I) Find two unbiased estimates for mean and variance.

(II) Estimate 95% confident intervals for average diameter of steel bars.

(III) What can you state about the diameter of the steel bars produced by the entity.

4. The company named A that involves in conducting researches regarding the Fruit cultivation declares that a particular germinated seed gives fruits within 54 days and the standard deviation as 4 days.

Another company named B declares that the germinated seeds of same kind of fruit gives fruit within 60 days and standard deviation as 6 days. A farmer has drawn random samples of seeds from each product and observed the duration spent to bear fruits. The average duration spent to bear Fruits and standard deviates are mentioned in the following table.

| Kind of seeds | Sample size | Duration of beariy fruits | |
|---------------|-------------|---------------------------|------------------------------|
| | | Mean \bar{x} | Standard Deviation $-\sigma$ |
| A | 64 | 58 | 3 |
| B | 49 | 59 | 5 |

Compute 95% confident intervals to estimate the difference of means of durations spent to bear fruits between the two kinds of seeds.

Marking Scheme

| Assessment Criteria | Assigning the Marks | | | |
|-------------------------------------|---------------------|---|---|---|
| | 1 | 2 | 3 | 4 |
| 1. Understanding the problem. | | | | |
| 2. Using assumptions appropriately. | | | | |
| 3. Accurate usage of formulas. | | | | |
| 4. Accuracy in calculations. | | | | |
| 5. Creativity in presentation. | | | | |